Name ______________________________

Differential Equations 2280

Midterm Exam 2

Exam Date: 1 April 2016 at 12:50pm

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Chapter 1)**

(a) [30%] Solve $y' + 2y = 3$.

(b) [30%] Solve $y' + 2xy = 0$.

(c) [40%] Solve $y' + y = 2e^x$.

Start your solution on this page.

2. (Chapter 3)

(a) [30%] Find the factors of the characteristic equation of a linear homogeneous constant coefficient differential equation of lowest order which has a particular solution

$$y(x) = 10 + 5xe^{x}\sin(x) + xe^{-x}.$$

(b) [40%] Determine for differential equation

$$\frac{d^2y}{dx^2} + \frac{dy}{dx} = x^3 + e^{-x} + \cos x$$

the shortest trial solution for y_p according to the method of undetermined coefficients. Do not evaluate the undetermined coefficients!

(c) [30%] Find the steady-state periodic solution for the spring-mass equation

$$\frac{d^2x}{dt^2} + 2\frac{dx}{dt} + 17x = 130\cos(t),$$

given a particular solution

$$x(t) = 4\mathrm{e}^{-t}\sin(4\,t) + 5\mathrm{e}^{-t}\cos(4\,t) + \sin(t) + 8\,\cos(t)$$

Start your solution on this page.

3. **(Laplace Theory)**

(a) [50%] Assume $f(t)$ is of exponential order. Find $f(t)$ in the relation

$$\left(\frac{d}{ds}\mathcal{L}(f(t))\right)\bigg|_{s\to(s+5)} = \frac{1}{s^2} + \frac{1}{(s+2)^2}.$$

(b) [50%] Find $\mathcal{L}(f)$ given $f(t) = e^{2t}\sin(t) + (e^t + e^{-t})^2$.

Start your solution on this page.

4. **(Laplace Theory)**

(a) [30%] The solution of $x'' + x' = 0$, $x(0) = 1$, $x'(0) = 0$ is $x(t) = 1$. Show the details in Laplace's Method for obtaining this answer.

(b) [40%] Solve the system $x' = x - y$, $y' = y + 2$, $x(0) = 0$, $y(0) = 0$ by Laplace's Method. Check the answer for y by the superposition shortcut for linear equations with constant coefficients.

(c) [30%] Find the Laplace transform of the convolution of $f(t) = e^t$ and $g(t) = t\cos t$.

Start your solution on this page.

5. (Systems of Differential Equations)

The eigenvalues of $A = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix}$ are 3, 5 with corresponding eigenvectors $\begin{pmatrix} -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix}$.

(a) [20%] Display the general solution of $\mathbf{u}' = A\mathbf{u}$ by the eigenanalysis method. Please use symbols c_1, c_2 for the constants that appear in the general solution.

(b) [50%] Display the details for solution of $\mathbf{u}' = A\mathbf{u}$, $\vec{u}(0) = \begin{pmatrix} 1 \\ -1 \end{pmatrix}$, according to the Cayley-Hamilton-Ziebur shortcut.

The scalar form of the system is

$$\begin{cases} x'(t) &= 4x(t) + y(t), \\ y'(t) &= x(t) + 4y(t), \\ x(0) &= 1, \\ y(0) &= -1. \end{cases}$$

Please observe that the initial conditions evaluate constants, therefore the answer for (b) does not contain symbols c_1, c_2.

(c) [30%] A fundamental matrix $\Phi(t)$ for $\mathbf{u}' = A\mathbf{u}$ is a 2×2 invertible matrix such that $\Phi'(t) = A\Phi(t)$. Using the answer from either (a) or (b), find one fundamental matrix $\Phi(t)$ for the system $\mathbf{u}' = \begin{pmatrix} 4 & 1 \\ 1 & 4 \end{pmatrix} \mathbf{u}$.

Start your solution on this page.