Name ________________________________

Differential Equations 2280

Midterm Exam 1

Exam Date: Friday, 26 February 2016 at 12:50pm

Instructions: This in-class exam is designed for 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. (Quadrature Equations)

(a) [40%] Solve $y' = \dfrac{2x^3}{1+x^2}$.

(b) [60%] Find the position $x(t)$ from the velocity model $\dfrac{d}{dt}(e^{-t}v(t)) = 2\,e^t$, $v(0) = 5$ and the position model $\dfrac{dx}{dt} = v(t)$, $x(2) = 2$.

Name. ___________________________________

2. (Classification of Equations)
The differential equation $y' = f(x,y)$ is defined to be **separable** provided $f(x,y) = F(x)G(y)$ for some functions F and G.

(a) [40%] The equation $y' + x(y+3) = ye^x + 3x$ is separable. Provide formulas for $F(x)$ and $G(y)$.

(b) [60%] Apply partial derivative tests to show that $y' = x + y$ is linear but not separable. Supply all details.

Name. ________________________________

3. (Solve a Separable Equation)
Given $(5y + 10)y' = \left(xe^{-x} + \sin(x)\cos(x)\right)(y^2 + 3y - 4)$.

Find a non-equilibrium solution in implicit form.
To save time, **do not solve** for y explicitly and **do not solve** for equilibrium solutions.

Name. ________________________________

4. (Linear Equations)

(a) [60%] Solve the linear model $2x'(t) = -64 + \dfrac{10}{3t+2}x(t)$, $x(0) = 32$. Show all integrating factor steps.

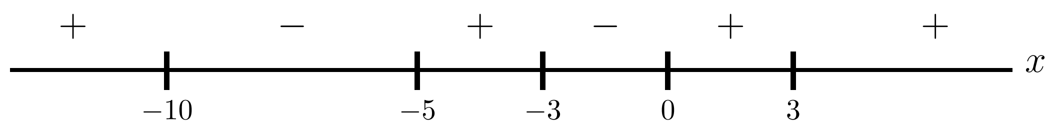
(b) [20%] Solve $\dfrac{dy}{dx} - (\cos(x))y = 0$ using the homogeneous linear equation shortcut.

(c) [20%] Solve $5\dfrac{dy}{dx} - 7y = 10$ using the superposition principle $y = y_h + y_p$ shortcut. Expected are answers for y_h and y_p.

Name. ________________________________

5. (Stability)

Assume an autonomous equation $x'(t) = f(x(t))$. Draw a phase portrait with at least 12 threaded curves, using the phase line diagram given below. Add these labels as appropriate: funnel, spout, node [neither spout nor funnel], stable, unstable.



Name. ___________________________________

6. (ch3)

Using Euler's theorem on Euler solution atoms and the characteristic equation for higher order constant-coefficient differential equations, solve (a), (b), (c).

(a) [40%] Find a constant coefficient differential equation $ay'' + by' + cy = 0$ which has particular solutions $-5e^{-x} + xe^{-x}$, $10e^{-x} + xe^{-x}$.

(b) [30%] Given characteristic equation $r(r-2)(r^3+4r)(r^2+2r+37) = 0$, solve the differential equation.

(c) [30%] Given $mx''(t) + cx'(t) + kx(t) = 0$, which represents an unforced damped spring-mass system. Assume $m = 4$, $c = 4$, $k = 129$. Classify the equation as over-damped, critically damped or under-damped. Illustrate in a spring-mass-dashpot drawing the assignment of physical constants m, c, k and the initial conditions $x(0) = 1$, $x'(0) = 0$.

Name. ________________________________

7. (ch3)

Determine for $y^{(3)} + y^{(2)} = x + 2e^{-x} + \sin x$ the corrected trial solution for y_p according to the method of undetermined coefficients. **Do not evaluate the undetermined coefficients**! The trial solution should be the one with fewest Euler solution atoms.