## Productive Economy

• Abstract

The goal is to show how to use linear algebra to find the relationship between input and output and decide whether the economy is productive. This question can be transformed into making a consumption matrix, which tells how much of each input goes into a unit of output and deciding whether it is a nonnegative matrix.

• Input and Output Information

Input and output information are available in the website. First, I transform US Economy Use Data 2008 into Use matrix and use the given Total Industry Output Vector, which has 65 rows. Then, I use Total Industry Output Vector to form a 65x65 diagonal matrix, which is the Total Output Matrix. I get the Total Output Matrix using the provided data: 

 $0.0.0.0.0.0.0.0.3.896*10^{5}, 0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0$  $0.0.0.0.0.0.0.0.0.0.0.2.929*10^{5}, 0.0.0.0.0.0.0.0.0.0.0.0$  $0.0.0.0.0.0.0.0.0.0.0.0.7.922*10^{5}, 0.0.0.0.0.0.0.0.0.0.0$ 0.0.0.0.0.0.0.0.0.0.0.0.8.583\*10<sup>5</sup>,0.0.0.0.0.0.0.0.0.0.0  $0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.1.554*10^{5}.0.0.0.0.0.0.0$ 0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.1.512\*10<sup>5</sup>,0.0.0.0.0.0] 0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.0.6.318\*10<sup>5</sup>.0.0.0.0.0] 

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• Consumption matrix

Consumption matrix tells how much of each input goes into a unit of output. For example, in an economy with 3 industrial sections:

[Chemical output]				[0.2 0.3 0.4][chemical use]		
[	food output	]	=	[0.4 0.4 0.1][ food	use]	
[	oil output	]		[ 0.5 0.1 0.3 ][ oil	use]	

In this equation, we can see that to produce a unit of chemicals requires 0.2 units of chemicals, 0.3 units of food, and 0.4 units of oil. This is what consumption matrix represents.

The consumption matrix A (i rows, j columns) can be got by rescaling column j of the Use Matrix by dividing by entry j of the Total Industry Output Vector so,

A=Use Matrix [i,j]/Total Industry Output Vector [j]

Also, according to consumption modeling the book, chapter 8.3, p is Total Industry Output Vector, A is the consumption matrix and y is demand. In this economy, some production p is consumed in the production process, and the other is left over to satisfy the outside demand, y. Ap will give the number of units of production that are consumed within the economy. p - Ap = y or  $p = (I-A)^{-1}y$ 

## • Solution

First, calculate the consumption matrix by the formula A=Use Matrix [i,j]/Total Industry Output Vector [j],  $1 \le i \le 65$  and  $1 \le j \le 65$ . Then, find the determinant of (I-A), which is not 0, so (I-A) is invertible and (I-A)<sup>-1</sup> exists. Also the demand y and the consumption matrix is nonnegative so (I-A)<sup>-1</sup> cannot be negative. Finally, calculate (I-A)<sup>-1</sup> and inspect every entries of this matrix. This is the way I solve the problem. Here is the result for (I-A)<sup>-1</sup>.

 $\begin{bmatrix} 1.202, 0.007, 0.0005, 0.0004, 0.001, 0.0003, 0.0023, 0.0011, 0.00038, 0.0023, 0.0028, 0.0016, 0.0008, 0.0009, 0.0010, 0.0007, 0.0009, 0.0019, 0.0021, 0.0019, 0.375, ] 0.0612, 0.0210, 0.0038, 0.0038, 0.0005, 0.0054, 0.0031, 0.0018, 0.0074, 0.0009, 0.0008, 0.0008, 0.0069, 0.0003, 0.0006, 0.0003, 0.0006, 0.0006, 0.0004, 0.0002, 0.0008, 0.0009, 0.0011, 0.0016, 0.0005, 0.0002, 0.0006, 0.0006, 0.0004, 0.0002, 0.0008, 0.0012, 0.0009, 0.0011, 0.0016, 0.0003, 0.0002, 0.0005, 0.0005, 0.0006, 0.0006, 0.0002, 0.0008, 0.0012, 0.0009, 0.0074, 0.0010, 0.0093, 0.0002, 0.0004, 0.0005, 0.0005, 0.0006, 0.0006, 0.0020, 0.0008, 0.0012, 0.0009, 0.0074, 0.0010, 0.0093, 0.008, 0.0012, 0.0009, 0.0074, 0.0010, 0.0093, 0.008, 0.0012, 0.0009, 0.0074, 0.0010, 0.0093, 0.008, 0.0011, 0.0016, 0.0005, 0.0005, 0.0005, 0.0006, 0.0011, 0.0012, 0.0064, 0.0116, 0.0026 \end{bmatrix}$ 

## .....

[0.0003,0.0002,0.0003,0.0003,0.0006,0.0001,0.0006,0.0001,0.0004,0.0009,0.0010,0.0020,0.001 1,

0.0008,0.0005,0.0008,0.0005,0.0008,0.0009,0.0008,0.0013,0.0008,0.0010,0.0.0008,0.0013,0.00 08,

0.0012,0.0013,0.0002,0.0008,0.0011,0.0005,0.0007,0.0002,0.0005,0.0003,00.0005,0.0001,0.000 5,

0.0017, 0.0008, 0.0007, 0.0004, 0.0002, 0.0002, 0.0006, 0.0002, 0.0012, 0.0004, 0.0009, 0.0006, 0.0011, 0.0033, 0.0004, 0.0016, 0.0007, 0.0090, 0.0007, 0.0007, 0.0007, 0.0041, 0.0012, 0.0010, 0.0010, 0.0039, 0.0027, 1.0346]

As we can see, all entries are positive, even if there are some negative numbers that are really close to 0. So, basically, the matrix  $(I-A)^{-1}$  is nonnegative.

There is also another way to decide whether the economy is productive, which is to find the largest eigenvalue  $\lambda_1$  of the consumption matrix, A.

If  $\lambda_1 > 1$ , then (I-A)<sup>-1</sup> has negative entries.

If  $\lambda_1 = 1$ , then (I-A)<sup>-1</sup> fails to exist.

If  $\lambda_1 < 1$ , then (I-A)<sup>-1</sup> is a nonnegative matrix.

• Conclusion

Most of entries of matrix (I-A)<sup>-1</sup> are positive and the rest of entries are close to 0, so the economic is productive. Also, this model is very useful in not only in analyzing the past economic data but also in predicting on what will happen as long as we know how use and consumption will change.

Reference

Strang, G (2009). Introduction to Linear Algebra 4<sup>th</sup> Edition, Wellesley-Cambridge Press http://www.math.utah.edu/~gustafso/s2012/2270/projects.html http://www.bea.gov/industry/io annual.htm