Benson Haglund

Dr. Gustafson

Math 2270 - Linear Algebra

May 4, 2016

Consumption Matrices and the Economy

In the early 1970's Wassily Leontief was doing research on how changes in one economic sector may have an effect on other sectors. In 1973, the Nobel Prize for Economics was awarded to Mr. Leontief for his work in input-output analysis. Mr. Leontief studied the characteristics of trade flow between the U.S. and other countries using input-output analysis. Mr. Leontief found "this country resorts to foreign trade in order to economize its capital and dispose of its surplus labor, rather than vice versa." Input-output analysis shows how dependent each economic sector is on all of the other sectors.

To begin the input-output analysis procedure, we must first create an input-output matrix for any given economy. The purpose of the input-output matrix is to see how goods from one industry are consumed in other industries. The producing sector of the economy is represented by the rows of the matrix, and the consuming sector of the economy is represented by the columns of the matrix. For example, the entry a_{ij} represents the amount of goods from sector i that sector j demands. The total internal demand for any economy is equal to the sum of the rows of the economies input-output matrix.

A structural matrix is a matrix that is constructed in terms of dollars rather than goods. This is helpful because it can give us information on the relative size of the industries in terms of currency so that we are better able to compare certain industries.

Structural matrices are created from the input-output matrix by finding the technical co-efficients. To find the Technical coefficients we need to divide the a_{ij} entry of the input-output matrix by a total industry output vector. The technical coefficients will show the quantity of the row product that will allow us to produce one unit of the column product. This allows us to know how much a certain industry used from the total industry output. We get a consumption matrix from the structural matrix that is composed of technical coefficients derived from the monetary value ratio.

Consumption matrices are important because we are able to see certain traits of the economy in which the system is operating. We are able to tell if a sector of the over-all economy is profitable by finding the sum of the columns. If the sum of the columns is less than one, this means that one unit of the goods costs less to produce and sell than the value that it sells for. Similarly, if the sum of the columns is greater than one, one unit of the goods costs more to produce and sell than the value that it sells for. There-fore, the producing economy is operating at a loss and is not productive.

To determine the productivity of the United States economy, we will create an input-output matrix using the data from the Bureau of Economic Analysis. The BEA has made a spreadsheet of the use of commodity by industry. They break they economy into fifteen sectors that produce goods or services. Those fifteen economic sectors are:

(1) Agriculture, Forestry, Fishing, and Hunting

(2) Mining

(3) Utilities

(4) Construction

(5) Manufacturing

(6) Wholesale Trade

(7) Retail Trade

(8) Transportation and Warehousing

(9) Information

(10) Finance, Insurance, Real Estate, Rental and Leasing

(11) Professional and Business Services

(12) Educational Services, Health Care, and Social Assistance

(13) Arts, Entertainment, Recreation, Accommodation, and Food Services

(14) Other Services, except government

(15) Government

The input-output matrix from this data looks like this:

$$A := \begin{bmatrix}
103993 & 105 & 0 & 1399 & 280940 & 1547 & 2244 & 108 & 1 & 35 & 1799 & 916 & 8214 & 82 & 2292 \\
2520 & 58402 & 43079 & 10742 & 560306 & 46 & 49 & 1580 & 339 & 4297 & 1563 & 435 & 1286 & 592 & 16944 \\
4955 & 4430 & 3076 & 2371 & 66293 & 5092 & 11171 & 6625 & 4414 & 71743 & 10765 & 25415 & 11448 & 4378 & 27741 \\
2884 & 5713 & 4749 & 158 & 16279 & 1577 & 2980 & 4683 & 2733 & 128332 & 2153 & 2970 & 2387 & 3884 & 72240 \\
87134 & 52838 & 25686 & 281090 & 2042570 & 45665 & 59431 & 198525 & 105380 & 61146 & 163507 & 190020 & 155655 & 63114 & 355582 \\
26447 & 8437 & 4813 & 47406 & 300870 & 41132 & 25790 & 37865 & 29959 & 15763 & 28149 & 39472 & 26013 & 10642 & 43856 \\
220 & 230 & 378 & 82613 & 13762 & 813 & 7650 & 5505 & 406 & 7181 & 2694 & 1687 & 7992 & 5409 & 595 \\
14259 & 13205 & 18308 & 20930 & 160719 & 66389 & 71188 & 119692 & 19398 & 29061 & 53577 & 2500 & 16158 & 6490 & 61579 \\
588 & 1225 & 1438 & 4530 & 26382 & 18091 & 20467 & 5818 & 241694 & 59498 & 79696 & 30260 & 10221 & 10078 & 73521 \\
21044 & 19499 & 10768 & 31460 & 85593 & 101708 & 147946 & 75213 & 68530 & 884455 & 246119 & 316319 & 93082 & 83370 & 134123 \\
6245 & 45803 & 19550 & 46229 & 445816 & 206347 & 166840 & 65236 & 149996 & 440548 & 566526 & 251468 & 141266 & 45125 & 249852 \\
495 & 0 & 148 & 20 & 85 & 1230 & 9106 & 75 & 927 & 50 & 817 & 27265 & 1975 & 2562 & 43850 \\
666 & 1197 & 2656 & 2444 & 23416 & 9312 & 6376 & 3398 & 35975 & 48182 & 69237 & 31235 & 29291 & 4222 & 31454 \\
1156 & 734 & 737 & 4900 & 18173 & 18919 & 12264 & 5413 & 10735 & 33889 & 35044 & 36953 & 12107 & 7354 & 25069 \\
47 & 6 & 454 & 26 & 6026 & 10472 & 6640 & 14479 & 2358 & 15152 & 9282 & 6726 & 6355 & 1824 & 8437
\end{bmatrix}$$

To turn the above input-output matrix into a consumption matrix, we need to obtain the technical coefficients. To get the technical coefficients we need to divide each column entry by the jth entry of the total industry output vector. The total industry output vector obtained from the BEA spreadsheet is as follows:

$$T := \begin{bmatrix} 488824 \\ 666540 \\ 417777 \\ 1204025 \\ 6178070 \\ 1576810 \\ 1550260 \\ 1068982 \\ 1510770 \\ 5298861 \\ 3336611 \\ 2407020 \\ 1186791 \\ 641527 \\ 3438166 \end{bmatrix}$$

After we divide the input-output matrix by our technical coefficients, we obtain a

consumption matrix C:

$$CI := \begin{bmatrix} 0.21274 & 0.00015753 & 0. & 0.0011619 & 0.045474 & 0.00098109 & 0.0014475 & 0.00010103 & 6.6191\ 10^{-7} & 0.0000066052 & 0.00053917 & 0.00038055 & 0.0069212 & 0.00012782 & 0.00066663 \\ 0.0051552 & 0.087620 & 0.10311 & 0.0089217 & 0.090693 & 0.000029173 & 0.000031608 & 0.0014780 & 0.00022439 & 0.00081093 & 0.00046844 & 0.00018072 & 0.0010836 & 0.00092280 & 0.0049282 \\ 0.010137 & 0.0066463 & 0.0073628 & 0.0019692 & 0.010730 & 0.0032293 & 0.0072059 & 0.0061975 & 0.0029217 & 0.013539 & 0.0032263 & 0.010559 & 0.0096462 & 0.0068243 & 0.0080685 \\ 0.0058999 & 0.0085711 & 0.011367 & 0.00013123 & 0.0026350 & 0.0010001 & 0.0019223 & 0.0043808 & 0.0018090 & 0.024219 & 0.00064527 & 0.0012339 & 0.0020113 & 0.0060543 & 0.021011 \\ 0.17825 & 0.079272 & 0.061483 & 0.23346 & 0.33062 & 0.028960 & 0.038336 & 0.18571 & 0.069753 & 0.011539 & 0.049004 & 0.078944 & 0.13116 & 0.098381 & 0.10342 \\ 0.054103 & 0.012658 & 0.011520 & 0.039373 & 0.048700 & 0.026086 & 0.016636 & 0.035422 & 0.019830 & 0.0029748 & 0.0084364 & 0.016399 & 0.021919 & 0.016589 & 0.012756 \\ 0.00045006 & 0.00034507 & 0.00090479 & 0.068614 & 0.0022276 & 0.00051560 & 0.0049347 & 0.0051498 & 0.00026874 & 0.0013552 & 0.00080741 & 0.00070087 & 0.0067341 & 0.0084314 & 0.00017306 \\ 0.029170 & 0.019811 & 0.043822 & 0.017383 & 0.026014 & 0.042103 & 0.045920 & 0.11197 & 0.012840 & 0.0054844 & 0.016057 & 0.0010386 & 0.013615 & 0.010116 & 0.017910 \\ 0.0012029 & 0.0018378 & 0.0034420 & 0.0037624 & 0.0042703 & 0.011473 & 0.013202 & 0.0054426 & 0.15998 & 0.011228 & 0.023885 & 0.012572 & 0.0086123 & 0.015709 & 0.021384 \\ 0.043050 & 0.029254 & 0.025775 & 0.026129 & 0.013854 & 0.064502 & 0.095433 & 0.070359 & 0.045361 & 0.16691 & 0.073763 & 0.13142 & 0.078432 & 0.12996 & 0.039010 \\ 0.012776 & 0.068718 & 0.046795 & 0.038395 & 0.072161 & 0.13086 & 0.10762 & 0.061026 & 0.099284 & 0.083140 & 0.16979 & 0.10447 & 0.11903 & 0.070340 & 0.072670 \\ 0.0010126 & 0. & 0.00035426 & 0.000016611 & 0.000013758 & 0.00078006 & 0.0058739 & 0.000070160 & 0.00061359 & 0.0000094360 & 0.00024486 & 0.011327 & 0.0016642 & 0.0039936 & 0.012754 \\ 0.0013625 & 0.0017958 & 0.0063575 & 0.0020299 & 0.0037902 & 0.0059056 & 0.0041129 & 0.0031787 & 0.023812 & 0.0090929 & 0.020751 & 0.012977 & 0.024681 & 0.0065812 & 0.0091485 \\ 0.0023649 & 0.0011012 & 0.0017641 & 0.0040697 & 0.0029415 & 0.011998 & 0.0079109 & 0.0050637 & 0.0071056 & 0.0063955 & 0.010503 & 0.015352 & 0.010201 & 0.011463 & 0.0072914 \\ 0.000096149 & 0.0000090017 & 0.0010867 & 0.000021594 & 0.00097539 & 0.0066413 & 0.0042832 & 0.013545 & 0.0015608 & 0.0028595 & 0.0027819 & 0.0027943 & 0.0053548 & 0.0028432 & 0.0024539 \end{bmatrix}$$

The columns and rows of the input-output matrix, and the consumption matrix

pertain to the economic sectors above in the same order. That is, column one is Agricul-

ture, Forestry, Fishing, and Hunting, column two is Mining etc.

In order to determine the productivity of the economy, we must add all of the

components of each column. If the sum of the entries in the column are less than one,

that means that the costs of goods and services for that sector of the economy cost less to produce and sell than the price that it sells for.

The sum of the entries for column one (Agriculture, Forestry, Fishing, and Hunting) was:

$$0.55777$$

Since this was less than one the economic sector "agriculture, forestry, fishing, and hunting" was productive during the year 2014.

The sum of the entries for column two (mining) was:

$$0.31781$$

Since this was less than one the economic sector of "mining" was productive in 2014.

The sum of column threes (Utilities) entries was:

$$0.32514$$

Since this sum was less than one, the economic sector of "Utilities" was productive during the year 2014.

The sum of the entries in column four (Construction) equaled:

$$0.44542$$

Since the sum of column fours entries was less than one, the economic sector of "Construction" was productive and made a profit in 2014.

The sum of the entries in column five (Manufacturing) equaled:

$$0.65510$$

As you can see, the sum of the entries for column five was less than one, so the economic sector of "Manufacturing" was productive in 2014.

The sum of the entries in column six (Wholesale Trade) equaled:

$$0.33506$$

Since column sixes sum was less than one, the economic sector "Wholesale Trade" was productive during 2014.

The sum of the entries in column seven (Retail Trade) equaled:

$$0.35485$$

Since this sum was less than one, "Retail Trade" had a productive year during 2014.

The sum of the entries in column eight (Transportation and Warehousing) equaled:

$$0.50910$$

Since this sum of the columns is less than one, the economic sector "Transportation and Warehousing" was productive in the year 2014 and made a profit.

The sum of the entries in column nine (Information) equaled:

$$0.44536$$

Since the sum of the entries for "Information" was less than one, the "Information" sector was productive for 2014.

The sum of the entries in column ten (Finance, Insurance, Real Estate, Rental and Leasing) equaled:

$$0.33957$$

Because the sum of the column was less than one, "Finance, Insurance, Real Estate, Rental and Leasing" made a profit in 2014.

The sum of the entries in column eleven (Professional and Business Services) equaled:

$$0.38089$$

The sum of the eleventh column was less than one so we can see that the economic sector of "Professional and Business Services" was productive for the year 2014.

The sum of the entries in column twelve (Educational Services, Health Care, and Social Assistance) equaled:

$$0.40035$$

As the sum of the column regarding "Educational Services, Health Care, and Social Assistance" was less than one, this economic sector was productive in 2014.

The sum of the entries in column thirteen (Arts, Entertainment, Recreation, Accommodation, and Food Services) equaled:

$$0.44104$$

Since the number is less than one, the economic sector of "Arts, Entertainment, Recreation, Accommodation, and Food Services" was productive and made a profit in the year 2014

The sum of the entries of column fourteen (Other Services, except government) equaled:

$$0.38833$$

Since the number is less than one, the economic sector of "other services, except Government" was productive and made a profit in the year 2014.

The sum of the entries of column fifteen (Government) equaled:

$$0.33363$$

Since the number is less than one, the economic sector of "Government" was productive and made a profit in the year 2014.

As you can see, the United States economy as a whole was productive during the year 2014.

In conclusion, consumption matrices and input-output analysis is an important tool when looking at economies. They can help an economist determine how productive an economy was based on the supply and demand of the economies various sectors. Because input-output analysis is linear, it allows easy, fast, and flexible computations of the changes in demand of an economy. Consumption matrices also work regardless of how big or small the economy that you are looking at is. This allows you to break the

economy into different sectors or even sub-sectors to determine the economic productivity. Lastly, Input-output analysis is an key tool that is often used in calculating important measures such as national GDP.