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Creating Fractal Art with a Computer

For this project I wanted to create fractal images using a computer. “A fractal is a pattern that is infinitely complex; that is, upon enlarging a portion of the fractal image, more complexity is revealed, and the same is true for a portion of that image, and so on ad infinitum.”

(math.ubc)Using linear algebra techniques a person can create their own fractals. This is done by performing linear transformations on some kind of image. There are two kinds of transformations that can be performed on an image to make fractals which are a similitude transformation and an affine transformation.

A similitude with a scale factor s is a mapping of \mathbb{R}^2 into \mathbb{R}^2 of the form

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = s \begin{bmatrix} \cos \theta & -\sin \theta \\ \sin \theta & \cos \theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \text{ where } s, \theta, e \text{ and } f \text{ are scalars (Klang)}$$

The similitude is a specific form of the more general formula for generating fractals which is the affine transformation, which looks like

$$T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = s \begin{bmatrix} a & c \\ b & d \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} e \\ f \end{bmatrix} \text{ where } a, b, c, d \text{ and } f \text{ are scalars (Klang)}$$

Using these two equations I was able to create three different fractal images. Two of the images had their equations provided and the third was an image that I created on my own.

Fractal Image One: Sierpinski Carpet

The first image that I created was sierpinski carpet. To create this fractal there are eight different linear transformations to be performed on a given image. For my image I chose a simple star that looks like the following. (Hirmes, J. and Eden)



The eight different transformation formulas to be performed in order to create sierpinski carpet are the following. (Klang)

$$1. \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 2/3 \end{bmatrix}$$

$$2. \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/3 \\ 2/3 \end{bmatrix}$$

$$3. \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix}$$

$$4. \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 1/3 \end{bmatrix}$$

$$5. \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2/3 \\ 1/3 \end{bmatrix}$$

$$6. \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix}$$

$$7. \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \end{bmatrix}$$

$$8. \quad T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \end{bmatrix}$$

After performing all of these transformations to the star, sierpinski carpet creates the following images.

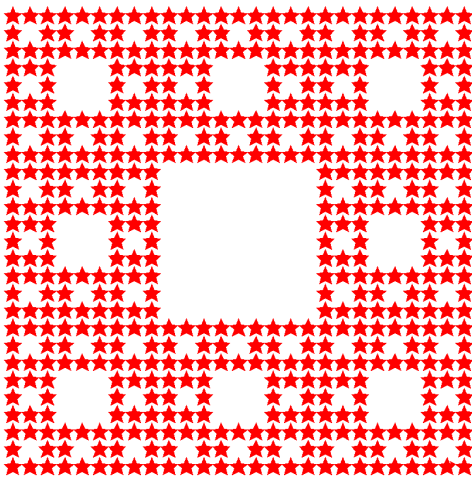
1 Transformation



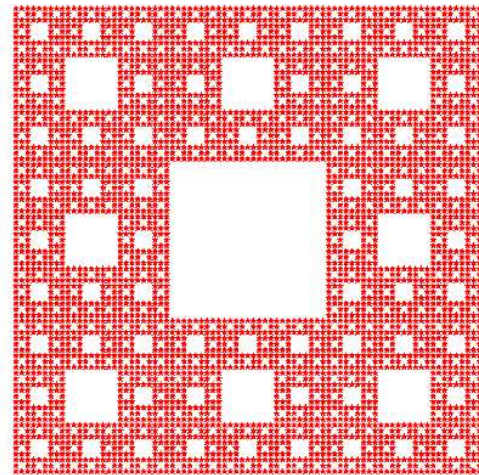
2 Transformations



3 Transformations



4 Transformations



Every time the transformation count increases the eight transformation formulas are performed to every single star in the previous transformation. So for 2 transformations all of the stars in first transformation are then transformed using the same eight formulas, then the third transformation transforms all the stars in the second transformation, and the fourth transformation transforms all of the stars in the third transformation.

Fractal Image Two: Koch Curve

The second image that I created was the Koch Curve. To create this fractal there are four different linear transformations to be performed on a line. The line is simply a straight line that looks like the following image.

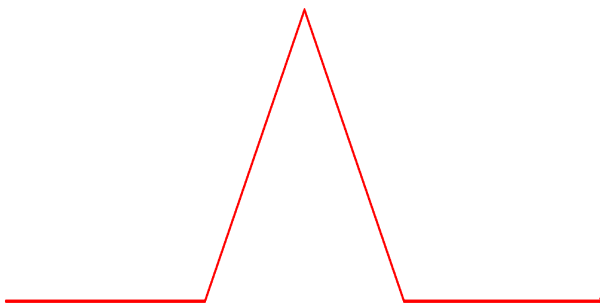


The four different transformation formulas to be performed in order to create the Koch Curve are the following. (Califano)

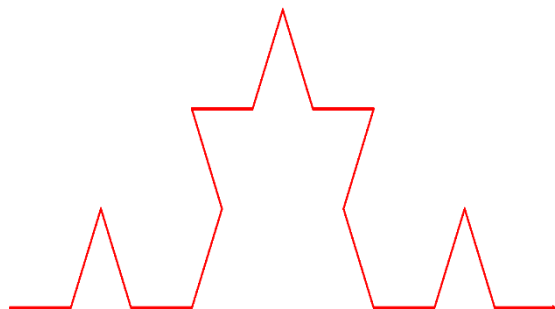
$$\begin{aligned} T1: & \begin{bmatrix} \frac{1}{3}\cos 0 & -\frac{1}{3}\sin 0 \\ \frac{1}{3}\sin 0 & \frac{1}{3}\cos 0 \end{bmatrix} * \begin{bmatrix} x_n \\ y_n \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \end{bmatrix} & T2: & \begin{bmatrix} \frac{1}{3}\cos(60) & -\frac{1}{3}\sin(60) \\ \frac{1}{3}\sin(60) & \frac{1}{3}\cos(60) \end{bmatrix} * \begin{bmatrix} x_n \\ y_n \end{bmatrix} + \begin{bmatrix} 1/3 \\ 0 \end{bmatrix} \\ T3: & \begin{bmatrix} \frac{1}{3}\cos(-60) & -\frac{1}{3}\sin(-60) \\ \frac{1}{3}\sin(-60) & \frac{1}{3}\cos(-60) \end{bmatrix} * \begin{bmatrix} x_n \\ y_n \end{bmatrix} + \begin{bmatrix} 1/2 \\ \sqrt{3}/6 \end{bmatrix} & T4: & \begin{bmatrix} \frac{1}{3}\cos 0 & -\frac{1}{3}\sin 0 \\ \frac{1}{3}\sin 0 & \frac{1}{3}\cos 0 \end{bmatrix} * \begin{bmatrix} x_n \\ y_n \end{bmatrix} + \begin{bmatrix} 2/3 \\ 0 \end{bmatrix} \end{aligned}$$

After performing all of these transformations to the line, the Koch Curve creates the following images.

1 Transformation

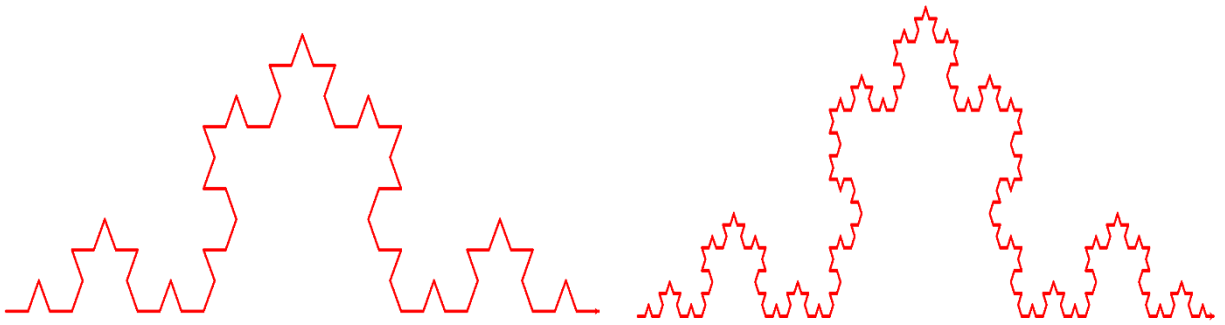


2 Transformations



3 Transformations

4 Transformations



Every time the transformation count increases the four transformation formulas are performed to every single line in the previous transformation. So for 2 transformations all of the lines in first transformation are then transformed using the same four formulas, then the third transformation transforms all the lines in the second transformation, and the fourth transformation transforms all of the lines in the third transformation.

Fractal Image Three: Personal Fractal

The third image that I created consisted of my own personal fractal. For this fractal I once again used the star image from the Sierpinski carpet fractal, but I used different transformation equations. I used six different transformations which are equations which were.

$$1) T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/4 \\ 27/80 \end{bmatrix} \quad 2) T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3/28 \\ 17/80 \end{bmatrix}$$

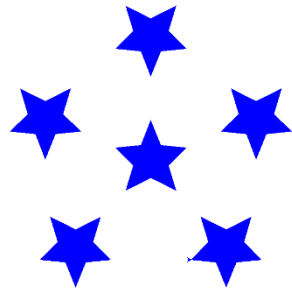
$$3) T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/4 \\ 459/560 \end{bmatrix} \quad 4) T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 1/4 \\ 97/80 \end{bmatrix}$$

$$5) T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 3/4 \\ 459/560 \end{bmatrix} \quad 6) T\left(\begin{bmatrix} x \\ y \end{bmatrix}\right) = \frac{1}{3}\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} 17/28 \\ 17/80 \end{bmatrix}$$

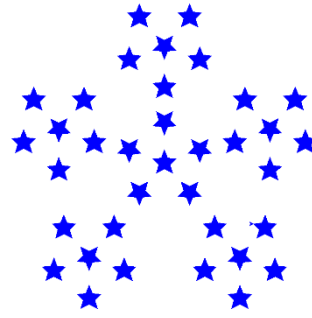
The matrix $\begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$ is used to flip the image through the x-axis. (Lay, Lay and McDonald 74)

After performing all of these transformations to the star, my fractal creates the following images.

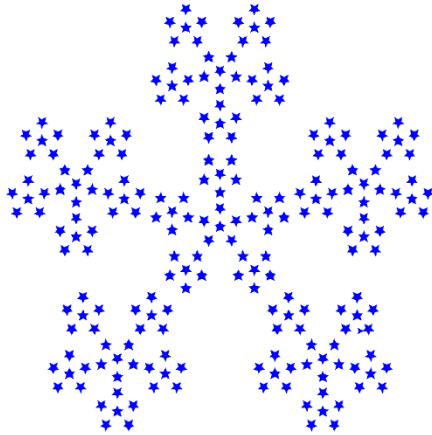
1 Transformation



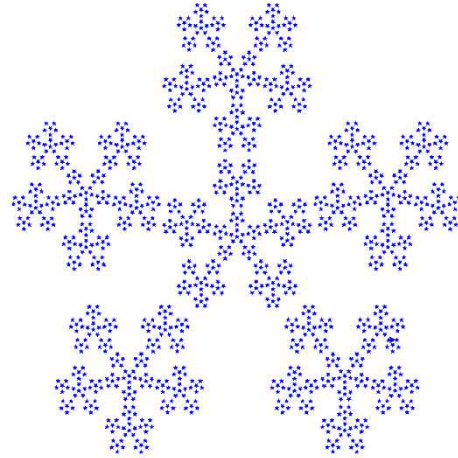
2 Transformations



3 Transformations



4 Transformations



Every time the transformation count increases the six transformation formulas are performed to every single star in the previous transformation. So for 2 transformations all of the stars in first transformation are then transformed using the same six formulas, then the third transformation transforms all the stars in the second transformation, and the fourth transformation transforms all of the stars in the third transformation.

For all three fractal images I used the python turtle library to create them and then took screen shots of the results. The python code for all three images has been provided.

Works Cited

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[03b/projects-03b/skinner/introduction.htm](http://www.math.ubc.ca/~cass/courses/m308-03b/projects-03b/skinner/introduction.htm)