Linear Algebra 2270-2

Due in Week 8

The eighth week starts the work from chapter 4. Here's the list of problems, followed by problem notes and a few answers.

Section 4.1. Exercises 5, 9, 11, 12, 16, 17, 19, 20, 21, 26

Section 4.2. Exercises 1, 2, 3, 11, 12, 17, 21, 27, 31

Section 4.3. Exercises 1, 6, 12, 17, 18, 21

Problem Notes

Issues for Strang's problems will be communicated here. For help, please send email, call 581-6879, or visit JWB 113.

Some Answers

4.1. Exercises 16, 21 have textbook answers.

4.1-11.

For A: The nullspace is spanned by $(-2, 1)$, the row space is spanned by $(1, 2)$. The column space is the line through $(1, 3)$ and $N(A^T)$ is the perpendicular line through $(3, -1)$.

For B: The nullspace of B is spanned by $(0, 1)$, the row space is spanned by $(1, 0)$. The column space and left nullspace are the same as for A.

4.1-17. If S is the subspace of $\mathcal{R}^3$ containing only the zero vector, then $S^\perp$ is $\mathcal{R}^3$. If S is spanned by $(1, 1, 1)$, then $S^\perp$ is the plane spanned by $(1, -1, 0)$ and $(1, 0, -1)$. If S is spanned by $(2, 0, 0)$ and $(0, 0, 3)$, then $S^\perp$ is the line spanned by $(0, 1, 0)$.

4.1-26. $A = \begin{pmatrix} 2 & 2 & -1 \\ -1 & 2 & 2 \\ 2 & -1 & 2 \end{pmatrix}$

This example shows a matrix with perpendicular columns. $A^T A = 9I$ is diagonal: $(A^T A)_{ij} =$ (column i of A) $\cdot$ (column j of A). When the columns are unit vectors, then $A^T A = I$.

4.2. Exercises 1, 3, 11, 21, 31 have textbook answers.

4.2-2.

(a) The projection of $b = (\cos\theta, \sin\theta)$ onto $a = (1, 0)$ is $p = (\cos\theta, 0)$.

(b) The projection of $b = (1, 1)$ onto $a = (1, -1)$ is $p = (0, 0)$ since $a^T b = 0$.

4.2-12. $P_1 = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$ = projection matrix onto the column space of A (the xy plane)

$P_2 = \begin{pmatrix} 0.5 & 0.5 & 0 \\ 0.5 & 0.5 & 0 \\ 0.0 & 0.0 & 1 \end{pmatrix}$ = Projection matrix onto the second column space. Certainly $(P_2)^2 = P_2$.

4.2-17. If $P^2 = P$ then $(I - P)^2 = (I - P)(I - P) = I^2 - PI - IP + P^2 = I - P$. When P projects onto the column space, then $I - P$ projects onto the left nullspace.

4.2-27. If $A^TAx = 0$ then Ax is in the nullspace of A^T. But Ax is always in the column space of A. To be in both of those perpendicular spaces, Ax must be zero. So A and A^TA have the same nullspace.

4.3. Exercises 1, 18, 21 have textbook answers.

4.3-6. $a = (1,1,1,1)$ and $b = (0,8,8,20)$ give $\hat{x} = \dfrac{a^Tb}{a^Ta} = 9$ and the projection is $\hat{x}a = p = (9,9,9,9)$. Then $e^Ta = (-9,-1,-1,11)^T(1,1,1,1) = 0$ and $\|e\| = \sqrt{204}$.

4.3-12.

(a) $a = (1,\ldots,1)$ has $a^Ta = m$, $a^Tb = b_1 + \cdots + b_m$. Therefore $\hat{x} = a^Tb/m$ is the mean of the b's

(b) $e = b - \hat{x}a$, $b = (1,2,b)$, $\|e\| = \sum_{i=1}^m (b_1 - \hat{x})^2$ = variance

(c) $p = (3,3,3)$, $e = (-2,-1,3)$, $p^Te = 0$. $P = \frac{1}{3}\begin{pmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{pmatrix}$.

4.3-17. $\begin{pmatrix} 1 & -1 \\ 1 & 1 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 7 \\ 7 \\ 21 \end{pmatrix}$. The solution $\vec{x} = \begin{pmatrix} 9 \\ 4 \end{pmatrix}$ comes from $\begin{pmatrix} 3 & 2 \\ 2 & 6 \end{pmatrix}\begin{pmatrix} C \\ D \end{pmatrix} = \begin{pmatrix} 35 \\ 42 \end{pmatrix}$.