Linear Algebra 2270-2

Due in Week 13

The thirteenth week finishes chapter 6 work. Extra exercises appear from Markov Matrices section 8.3. Here's the list of problems, followed by problem notes and answers.

Problem week13-1. Find the eigenvalues of this Markov matrix (their sum is the trace): $A = \begin{pmatrix} .90 & .15 \\ .10 & .85 \end{pmatrix}$.

What is the steady state eigenvector for the eigenvalue $\lambda_1 = 1$? See Exercise 8.3-1.

Problem week13-2. Prove that the square of a Markov matrix is also a Markov matrix. See Exercise 8.3-9.

Problem week13-3. If A is a Markov matrix, then does $I + A + A^2 + \cdots$ add up to the *resolvent* $(A - I)^{-1}$? See Exercise 8.3-17.

Section 6.6. Exercises 3, 17, 20

Section 6.7. Exercises 1, 4, 5, 6

Problem Notes

Issues for Strang's problems will be communicated here. For help, please send email, call 581-6879, or visit JWB 113.

Some Answers

Problem week13-1. Eigenvalues $\lambda = 1, 0.75$; $(A - I)x = 0$ gives the steady state $x = (.6, .4)$ with $Ax = x$.

Problem week13-2. M^2 is still nonnegative; multiply M on the left by $y = [1, \ldots, 1]$ (all ones) to obtain $yM = y$. Then multiply $yM = y$ on the right by M to find $yM^2 = y$, which implies that the columns of M^2 add to 1.

Problem week13-3. No, A has an eigenvalue $\lambda = 1$ and $(I - A)^{-1}$ does not exist.

6.6. Exercise 17 has a textbook answer.

6.6-3. $B = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix} = M^{-1}AM$;

$B = \begin{pmatrix} 1 & -1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$;

$B = \begin{pmatrix} 4 & 3 \\ 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}^{-1} \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$.

6.6-20. (a) $A = M^{-1}BM$ implies $A^2 = AA = M^{-1}B^2M$. So A^2 is similar to B^2. (b) A^2 equals $(-A)^2$ but A may not be similar to $-B$ (it could be!). (c) $\begin{pmatrix} 3 & 1 \\ 0 & 4 \end{pmatrix}$ is diagonalizable to $\begin{pmatrix} 3 & 0 \\ 0 & 4 \end{pmatrix}$ because $\lambda_1 \neq \lambda_2$, so these matrices are similar. (d) $\begin{pmatrix} 3 & 1 \\ 0 & 3 \end{pmatrix}$ has only one eigenvector, so it is not diagonalizable (e) PAP^T is similar to A.

6.7. Exercises 1, 4, 5 have textbook answers.

6.7-6. $AA^T = \begin{pmatrix} 2 & 1 \\ 1 & 2 \end{pmatrix}$ has $\sigma_1^2 = 3$ with $u_1 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $\sigma_2^2 = 1$ with $u_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$.

$A^T A = \begin{pmatrix} 1 & 1 & 0 \\ 1 & 271 & \\ 0 & 1 & 1 \end{pmatrix}$ has $\sigma_1^2 = 3$ with $v_1 = \frac{1}{\sqrt{6}} \begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix}$, $\sigma_2^2 = 1$ with $v_2 = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}$ and $v_3 = \frac{1}{\sqrt{3}} \begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$.

Then

$$\begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \end{pmatrix} = \mathbf{aug}(u_1, u_2) \begin{pmatrix} \sqrt{3} & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \mathbf{aug}(v_1, v_2, v_3)^T$$