Linear Algebra 2270-2

Due in Week 11

The eleventh week starts eigenanalysis chapter 6. Here's the list of problems, followed by problem notes and answers.

Section 6.1. Exercises 1, 2, 3, 4, 5, 6, 9, 16, 25, 32

Section 6.2. Exercises 4, 11, 12, 15, 16, 18, 20, 26

Problem Notes

Issues for Strang's problems will be communicated here. For help, please send email, call 581-6879, or visit JWB 113.

Some Answers

6.1. Exercises 1, 3, 6, 16, 32 have textbook answers.

6.1-2. A has $\lambda_1 = -1$ and $\lambda_2 = 5$ with eigenvectors $x_1 = (-2, 1)$ and $x_2 = (1, 1)$. The matrix $A + I$ has the same eigenvectors, with eigenvalues increased by 1 to 0 and 6. That zero eigenvalue correctly indicates that $A + I$ is singular.

6.1-4. A has $\lambda_1 = -3$ and $\lambda_2 = 2$ (check trace $= -1$ and determinant $= -6$) with $x_1 = (3, -2)$ and $x_2 = (1, 1)$. A^2 has the same eigenvectors as A, with eigenvalues $\lambda_1^2 = 9$ and $\lambda_2^2 = 4$.

6.1-5. A and B have eigenvalues 1 and 3. $A + B$ has $\lambda_1 = 3$, $\lambda_2 = 5$. Eigenvalues of $A + B$ are not equal to eigenvalues of A plus eigenvalues of B.

6.1-9. (a) Multiply by A: $A(Ax) = A(\lambda x) = \lambda Ax$ gives $A^2x = \lambda^2 x$ (b) Multiply by A^{-1}: $x = A^{-1}Ax = A^{-1}(\lambda x) = \lambda A^{-1}x$ gives $A^{-1}x = \frac{1}{\lambda}x$ (c) Add $Ix = x$: $(A + I)x = (\lambda + 1)x$.

6.1-25. With the same n eigenpairs (λ_i, x_i), then $x = c_1x_1 + \cdots + c_nx_n$ implies $Ax = c1\lambda_1x_1 + \cdots + c_n\lambda_nx_n$ and $Bx = c1\lambda_1x_1 + \cdots + c_n\lambda_nx_n$, therefore $Ax = Bx$ for all vectors x, which implies $A = B$.

6.2. Exercises 4, 12, 15, 26 have textbook answers.

6.2-11. (a) True (no zero eigenvalues) (b) False (repeated $\lambda = 2$ may have only one line of eigenvectors) (c) False (repeated λ may have a full set of eigenvectors).

6.2-16. $\Lambda = \begin{pmatrix} 1 & 0 \\ 0 & 0.2 \end{pmatrix}$, $S = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$, $\Lambda^k \to \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, $S\Lambda^kS^{-1} \to \frac{1}{2}\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$, which is the steady state.

6.2-18. $A^k = \frac{1}{2}\begin{pmatrix} 1+3^k & 1-3^k \\ 1-3^k & 1+3^k \end{pmatrix}$

6.2-20. This proof works when A is diagonalizable, $A = S\Lambda S^{-1}$:

$$\det(A) = \det(S)\det(\Lambda)\det(S^{-1}) = \det(\Lambda) = \lambda_1 \cdots \lambda_n$$