The fifth week finishes the work from chapter 1 and starts chapter 2. Here’s the list of problems, followed by a few answers.

**Section 1.9.** Exercises 15, 25, 31, 39

**Section 2.1.** Exercises 13, 18, 23, 28, 29

**Section 2.2.** Exercises 11, 23, 24, 35

**Section 2.3.** Exercises 2.3: 7, 13, 14, 17, 19, 20, 21, 33, 35

**Some Answers**

**2.1-18:** The first two columns of $AB$ are $Ab_1$ and $Ab_2$. They are equal since $b_1$ and $b_2$ are equal.

**2.1-28:** Since the inner product $u^Tv$ is a real number, it equals its transpose. That is, $u^Tv = (u^Tv)^T = v^T(u^Tv) = v^Tu$, by Theorem 3(d) regarding the transpose of a product of matrices and by Theorem 3(a). The outer product $uv^T$ is an $nn$ matrix. By Theorem 3, $(uv^T)^T = (v^Tu)^T = vu^T$.

**2.2-24:** If the equation $Ax = b$ has a solution for each $b$ in $\mathcal{R}^n$, then $A$ has a pivot position in each row, by Theorem 4 in Section 1.4. Since $A$ is square, then the pivots must be on the diagonal of $A$. It follows that $A$ is row equivalent to $I_n$. By Theorem 7, matrix $A$ is invertible.

**2.3-14:** If $A$ is lower triangular with nonzero entries on the diagonal, then these $n$ diagonal entries can be used as pivots to produce zeros below the diagonal. Thus $A$ has $n$ pivots and so is invertible, by the Invertible Matrix Theorem. If one of the diagonal entries in $A$ is zero, then $A$ will have fewer than $n$ pivots and hence be singular.

**2.3-20:** By the box following the Invertible Matrix Theorem, $E$ and $F$ are invertible and are inverses. So $FE = I = EF$, and therefore matrices $E$ and $F$ commute.