Linear Algebra 2270-2
Due in Week 5

The fifth week finishes the work from chapter 1 and starts chapter 2. Here's the list of problems, followed by a few answers.

Section 1.9. Exercises 15, 25, 31, 39

Section 2.1. Exercises 13, 18, 23, 28, 29

Section 2.2. Exercises 11, 23, 24, 35

Section 2.3. Exercises 2.3: 7, 13, 14, 17, 19, 20, 21, 33, 35

Some Answers

2.1-18: The first two columns of AB are Ab_1 and Ab_2. They are equal since b_1 and b_2 are equal.

2.1-28: Since the inner product u^Tv is a real number, it equals its transpose. That is, $u^Tv = (u^Tv)^T = v^T(u^T)^T = v^Tu$, by Theorem 3(d) regarding the transpose of a product of matrices and by Theorem 3(a). The outer product uv^T is an nn matrix. By Theorem 3, $(uv^T)^T = (v^T)^Tu^T = vu^T$.

2.2-24: If the equation $Ax = b$ has a solution for each b in $\mathcal{R}^n$, then A has a pivot position in each row, by Theorem 4 in Section 1.4. Since A is square, then the pivots must be on the diagonal of A. It follows that A is row equivalent to I_n . By Theorem 7, matrix A is invertible.

2.3-14: If A is lower triangular with nonzero entries on the diagonal, then these n diagonal entries can be used as pivots to produce zeros below the diagonal. Thus A has n pivots and so is invertible, by the Invertible Matrix Theorem. If one of the diagonal entries in A is zero, then A will have fewer than n pivots and hence be singular.

2.3-20: By the box following the Invertible Matrix Theorem, E and F are invertible and are inverses. So $FE = I = EF$, and therefore matrices E and F commute.