MATH 2270-2 Exam 1 Spring 2016

1. (10 points)

(a) Give a counter example or explain why it is true. If A and B are $n \times n$ invertible, and C^T denotes the transpose of a matrix C, then $((A+B)^{-1})^T = (B^T)^{-1} + (A^T)^{-1}$.

(b) Give a counter example or explain why it is true. If A is a square matrix and $A^T A = I$, then both A and A^T are invertible.

2. (10 points) Let A be a 3×4 matrix. Find the elimination matrix E which under left multiplication against A performs both (1) and (2) with one matrix multiply.

(1) Replace Row 2 of A with Row 2 minus Row 1.

(2) Replace Row 3 of A by Row 3 minus 5 times Row 2.

3. (30 points) Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & -b & c \\ 1 & c & a \\ 2 & -b+c & -a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a \\ -a \\ -a \end{pmatrix}$$

Use techniques learned in this course to briefly explain the following facts. Only write what is needed to justify a statement.

(a). The system has a unique solution for $(b+c)(2a+c) \neq 0$.

(b). The system has no solution if $2a+c=0$ and $a \neq 0$ (don't explain the other possibilities).

(c). The system has infinitely many solutions if $a = c = 0$ (don't explain the other possibilities).

4. (20 points) Definition. *Vectors* $\vec{v}_1, \ldots, \vec{v}_k$ *are called* **independent** *provided solving the equation* $c_1\vec{v}_1 + \cdots + c_k\vec{v}_k = \vec{0}$ *for constants* $c_1, \ldots, c_k$ *has the unique solution* $c_1 = \cdots = c_k = 0$. *Otherwise the vectors are called* **dependent**.

Find a largest set of independent vectors from the following set of vectors, using the definition of independence (above). You may use the Pivot Theorem without explanation. Any independence test from a reference textbook may be used, provided you state the test.

$$\begin{pmatrix}1\\0\\0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\2\\0\\0\\0\end{pmatrix}, \begin{pmatrix}0\\0\\0\\0\\0\end{pmatrix}, \begin{pmatrix}1\\0\\0\\0\\1\end{pmatrix}, \begin{pmatrix}3\\2\\0\\0\\1\end{pmatrix}, \begin{pmatrix}2\\0\\0\\0\\1\end{pmatrix}$$

5. (20 points) Find the vector general solution $\vec{x}$ to the equation $A\vec{x} = \vec{b}$ for

$$A = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 3 & 0 & 1 & 0 \\ 4 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 4 \\ 0 \end{pmatrix}$$

6. (20 points) Determinant problem, chapter 3. Parts reduced on Exam 1.

(a) [10%] True or False? The value of a determinant is multiplied by -1 when two columns are swapped.

(b) [10%] True or False? The determinant of two times the $n \times n$ identity matrix is 2.

(c) [30%] Assume given 3×3 matrices A, B. Suppose $E_3 E_2 E_1 A = BA^2$ and E_1, E_2, E_3 are elementary matrices representing respectively a multiply by 3, a swap and a combination. Assume $\det(B) = 3$. Find all possible values of $\det(-2A)$.

(d) [20%] Determine all values of x for which $(I + 2C)^{-1}$ fails to exist, where I is the 3×3 identity and $C = \begin{pmatrix} 2 & x & 1 \\ x & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}$.

(e) [30%] Let symbols a, b, c denote constants and define

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ a & b & 0 & \frac{1}{2} \\ 1 & c & 1 & \frac{1}{2} \end{pmatrix}$$

Apply the adjugate [adjoint] formula for the inverse

$$A^{-1} = \frac{\mathbf{adj}(A)}{|A|}$$

to find the value of the entry in row 4, column 1 of A^{-1}.

End Exam 1.