

MATH 2270-2 Sample Exam 1 Spring 2016

NAME (please print): _____

No books or notes. No electronic devices, please.

The questions have credits 10 to 30, which reflects the time required to write the solution.

If you must write a solution out of order or on the back side, then supply a road map.

The actual exam will have the same number of problems, identical problem types, on exactly the same topics. Covered on the exam are chapters 1, 2, 3 from the 2270 textbook.

1. (10 points)

(a) Give a counter example or explain why it is true. If A and B are $n \times n$ invertible, and C^T denotes the transpose of a matrix C , then $(AB^{-1})^T = (B^T)^{-1}A^T$.

(b) Give a counter example or explain why it is true. If square matrices A and B satisfy $AB = I$, then $BA = I$ and $A^T B^T = I$.

2. (10 points) Let A be a 3×4 matrix. Find the elimination matrix E which under left multiplication against A performs both (1) and (2) with one matrix multiply.

(1) Replace Row 2 of A with Row 2 minus Row 3.

(2) Replace Row 3 of A by Row 3 minus 4 times Row 1.

3. (30 points) Let a , b and c denote constants and consider the system of equations

$$\begin{pmatrix} 1 & b & c \\ 1 & c & -a \\ 2 & b+c & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} -a \\ a \\ a \end{pmatrix}$$

Use techniques learned in this course to briefly explain the following facts. Only write what is needed to justify a statement.

- (a). The system has a unique solution for $(c - b)(2a - c) \neq 0$.
- (b). The system has no solution if $c = 2a$ and $a \neq 0$ (don't explain the other possibilities).
- (c). The system has infinitely many solutions if $a = b = c = 0$ (don't explain the other possibilities).

Definition. Vectors $\vec{v}_1, \dots, \vec{v}_k$ are called **independent** provided solving the equation $c_1\vec{v}_1 + \dots + c_k\vec{v}_k = \vec{0}$ for constants c_1, \dots, c_k has the unique solution $c_1 = \dots = c_k = 0$. Otherwise the vectors are called **dependent**.

4. (20 points) Classify the following sets of vectors as Independent or Dependent, using the Pivot Theorem or the definition of independence (above).

$$\text{Set 1: } \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 2 \\ 2 \\ 0 \end{pmatrix}$$

$$\text{Set 2: } \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}$$

5. (20 points) Find the vector general solution \vec{x} to the equation $A\vec{x} = \vec{b}$ for

$$A = \begin{pmatrix} 1 & 0 & 0 & 4 \\ 3 & 0 & 1 & 0 \\ 4 & 0 & 0 & 1 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} 0 \\ 4 \\ 0 \end{pmatrix}$$

6. (20 points) Determinant problem, chapter 3. Parts reduced on Exam 1.

(a) [10%] True or False? The value of a determinant is the product of the diagonal elements.

(b) [10%] True or False? The determinant of the negative of the $n \times n$ identity matrix is -1 .

(c) [30%] Assume given 3×3 matrices A, B . Suppose $E_2 E_1 A^2 = AB$ and E_1, E_2 are elementary matrices representing respectively a combination and a multiply by 3. Assume $\det(B) = 27$. Let $C = -A$. Find all possible values of $\det(C)$.

(d) [20%] Determine all values of x for which $(2I + C)^{-1}$ fails to exist, where I is the 3×3

identity and $C = \begin{pmatrix} 2 & x & -1 \\ 3x & 0 & 1 \\ 1 & 0 & -1 \end{pmatrix}$.

(e) [30%] Let symbols a, b, c denote constants and define

$$A = \begin{pmatrix} 1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ a & b & 0 & 1 \\ 1 & c & 1 & \frac{1}{2} \end{pmatrix}$$

Apply the adjugate [adjoint] formula for the inverse

$$A^{-1} = \frac{\mathbf{adj}(A)}{|A|}$$

to find the value of the entry in row 4, column 2 of A^{-1} .

End Exam 1.