MATH 2270-2 Final Exam Study Guide Spring 2012

The Final Exam is Monday, April 30, 2012, from 1:00pm to 3:00pm, in AEB 320. The exam covers Chapters 1-7 of the text.

Basic Topics

Vector norm, dot product, angle between two vectors, orthogonality, vector and matrix algebra, solving systems of linear equations, Gaussian Elimination, Gauss-Jordan Elimination and pivots, reduced row echelon form, LU-decomposition, elementary matrices, rank, nullity, determinants, inverse of a matrix, Cramer's rule, adjugate identity, cofactor expansion, the Four Rules to compute any determinant, elementary matrices and determinants, nullspace, image, row space, column space, transpose, left subspace, vector subspaces, basis and dimension, linear independence, pivot theorem, determinant test, rank test, Wronskian test, sample test, the four fundamental subspaces, shadow projection, reflection and complex conjugate, orthogonal matrices, orthogonal projection onto a subspace, $S^{\perp}$ subspace, least squares approximation, Near Point Theorem, Gram-Schmidt process, QR-factorization, eigenanalysis, eigenvalues, eigenvectors, eigenpairs, Fourier's method, diagonalization, positive-definite matrices, similar matrices, Cayley-Hamilton Theorem, differential systems, Picard iteration, Picard-Lindelöf existence-uniqueness theorem, exponential matrix, Jordan's Theorem, Spectral theorem, Jordan block, Jordan Canonical form, Singular Value Decomposition, linear transformations, shear, rotation, projection, reflection, scale, one-to-one maps, onto maps, homeomorphism, coordinates of a vector relative to a basis, matrix pseudo-inverse.

Problems

There will be 60-70 percent homework-type problems. A sample exam appears on the course web site, which details the problem types and the amount of writing expected during the 2-hour final exam.

Applied topics

Graphs and networks, Kirchhoff's Laws, Least Squares, Fourier series, Markov matrices, Google Algorithm. These topics appeared on maple labs and midterms, but **they will not appear on the final exam**.

Essay questions

The questions will cover the highlights of the course. During the exam, you will write down your answer from memory alone. Notes and books are not allowed.

Expect to answer one or more of these questions:

1. State the Three Possibilities for a linear system $A\vec{x} = \vec{b}$. One possibility is a unique solution $\vec{x}$.

2. State the fundamental theorem on frame sequences. For instance, a sequence of three swap, combo, multiply operations can be compressed to a matrix multiply equation $E_3E_2E_1A = B$.

3. State the Rank-Nullity theorem, which is the fundamental relation for lead and free variables.

4. State the product theorem for determinants, the adjugate identity for the inverse, and Cramer's Rule.

5. Give an example for the Last Frame Algorithm, which writes out the solution of $A\vec{x} = \vec{b}$ in the case of infinitely many solutions.

6. State the Fundamental Theorem of Linear Algebra, Parts 1 and 2. Write a single sentence which summarizes this complex statement, using the homogeneous equation $A\vec{x} = \vec{0}$, the row space and the nullspace.

7. Display the normal equations for the theory of least squares and state the Near Point theorem.

8. State the Cayley-Hamilton theorem. Display an example for 2×2 matrices.

9. State the Spectral Theorem for symmetric matrices.

10. State Jordan's Theorem: a square matrix is similar to a triangular matrix.

10. Define the Jordan Canonical Form J. Then state Jordan's theorem about $A = PJP^{-1}$.

11. State the theorem for the Singular Value Decomposition (SVD) of A. Then display the equation for the pseudo-inverse A^+.

12. Explain the relationship between the four fundamental subspaces on the cover of Strang's book and the orthogonal matrices U, V of the singular value decomposition.