Frame Sequences with Symbol k

Math 2250 Spring 2010

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Example: Three Possibilities with Symbol k

Determine all values of the symbol k such that the system below has one of the **Three Possibilities** (1) *No solution*, (2) *Infinitely many solutions* or (3) *A unique solution*. Display all solutions found.

$$\begin{array}{rcrcl} x & + & ky & = & 2, \\ (2-k)x & + & y & = & 3. \end{array}$$

The Three Possibilities are detected by (1) A signal equation "$0 = 1$," (2) One or more free variables, (3) Zero free variables.

The solution of this problem involves construction of perhaps three frame sequences, the last frame of each resulting in one of the three possibilities (1), (2), (3).

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Details

A portion of the frame sequence is constructed, as follows.

$$\begin{array}{rcrcr} x & + & ky & = & 2, \\ (2-k)x & + & y & = & 3. \end{array}$$

Frame 1.

Original system.

$$\begin{array}{rcrcr} x & + & ky & = & 2, \\ & & [1+k(k-2)]y & = & 2(k-2)+3. \end{array}$$

Frame 2.

`combo(1,2,k-2)`

$$\begin{array}{rcrcr} x & + & ky & = & 2, \\ & & (k-1)^2 y & = & 2k-1. \end{array}$$

Frame 3.

Simplify.

The three expected frame sequences share these initial frames. At this point, we identify the values of k that split off into the three possibilities.

Three Possibilities

$$\begin{array}{rcrcr} x & + & ky & = & 2, \\ & & (k-1)^2 y & = & 2k-1. \end{array}$$

Frame 3.

Simplify.

There will be a signal equation if the second equation of Frame 3 has no variables, but the resulting equation is not "$0 = 0$." This happens exactly for $k = 1$. The resulting signal equation is "$0 = 1$." We conclude that one of the three frame sequences terminates with the *no solution case*. This frame sequence corresponds to $k = 1$.

Otherwise, $k \neq 1$. For these values of k, there are zero free variables, which implies a unique solution. A by-product of the analysis is that the *infinitely many solutions* case never occurs!

The conclusion

The initially expected three frame sequences reduce to two frame sequences. One sequence gives no solution and the other sequence gives a unique solution.

The three answers:

(1) No solution occurs only for $k = 1$*.*
(2) Infinitely many solutions occurs for no value of k*.*
(3) A unique solution occurs for $k \neq 1$*. The solution is then*

$$x = 2 - \frac{k(2k-1)}{(k-1)^2},$$

$$y = \frac{(2k-1)}{(k-1)^2}.$$