Math 2280
Maple Lab 8: Earthquake project
S2015

Name ________________________________ Class Time ________

Project 8. Solve problems L8-1 to L8-5. The problem headers:

_______ PROBLEM L8.1. EARTHQUAKE MODEL FOR A BUILDING.
_______ PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
_______ PROBLEM L8.3. UNDETERMINED COEFFICIENTS STEADY-STATE SOL
_______ PROBLEM L8.4. PRACTICAL RESONANCE.
_______ PROBLEM L8.5. EARTHQUAKE DAMAGE.

FIVE FLOOR Model.
Refer to the textbook of Edwards-Penney, section 5.3 Application (after
the section 5.3 exercises).
Consider a building with five floors each weighing 50 tons. Each floor
corresponds to a restoring Hooke's force with constant k=5 tons/foot.
Assume that ground vibrations from the earthquake are modeled by
(1/4)cos(wt) with period T=2*Pi/w.

PROBLEM L8-1. BUILDING MODEL FOR AN EARTHQUAKE.
Model the 5-floor problem in Maple.
Define the 5 by 5 mass matrix M and Hooke's matrix K for this system
and convert Mx''=Kx into the system x''=Ax where A is defined by
textbook equation (1), section 5.3 Application.
Sanity check: Mass m=3125, and the 5x5 matrix contains fraction 16/5.

Then find the eigenvalues of the matrix A to six digits, using the
Maple command "linalg[eigenvals](A)."
Sanity check: All six eigenvalues should be negative.

# Sample Maple code for a model with 4 floors.
# Use maple help to learn about evalf and eigenvals.
# A:=matrix([ [-20,10,0,0], [10,-20,10,0],
# [0,10,-20,10],[0,0,10,-10]]);
# with(linalg): evalf(eigenvals(A));

# Problem L8.1
# Define k, m and the 5x5 matrix A.
# with(linalg): evalf(eigenvals(A));

PROBLEM L8-2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
Refer to figure 5.3.17 in Edwards-Penney.

Find the natural angular frequencies omega=sqrt(-lambda) for the
five story building and also the corresponding periods
2Pi/omega, accurate to six digits. Display the answers in a table.
Compare with answers in Figure 5.3.17 (actually a table), for the 7-story case.

# Sample code for a 4x3 table, 4-story building.
# Use maple help to learn about nops and printf.
Problem L8-2

Define \( b := 0.25 \cdot w \cdot w \cdot \text{vector}([1,1,1,1,1]) \): in Maple and find the vector \( c \) in the undetermined coefficients solution \( x(t) = \cos(wt)c \). Vector \( c \) depends on \( w \). As outlined in the textbook, vector \( c \) can be found by solving the linear algebra problem \(-w^2 c = Ac + b\); see equation (32), section 5.3. Don't print \( c \), as it is too complex; instead, print \( c[1] \) as an illustration.

Sample code for defining \( b \) and \( A \), then solve for \( c \), 4-floor case.

Define \( w := 'w': u := w \cdot w: b := 0.25 \cdot u \cdot \text{vector}([1,1,1,1,1]) \):

\[
A := \text{matrix}([[ -20,10,0,0], [10,-20,10,0], [0,10,-20,10], [0,0,10,-10]]);
\]

\[
\text{Au} := \text{evalm}(A + u \cdot \text{diag}(1,1,1,1,1));
\]

\[
c := \text{linsolve}(\text{Au},-b);
\]

\[
\text{evalf}(c[1],2);
\]

Problem L8-3

Define \( w \), \( u \), \( b \), \( A \), \( \text{Au} \), \( c \)

\[
\text{evalf}(c[1],2);
\]

Problem L8-4. Practical Resonance.

Consider the forced equation \( x'=Ax+\cos(wt)b \) of L8-3 above with \( b := 0.25 \cdot u \cdot w \cdot \text{vector}([1,1,1,1,1]) \).

Practical resonance can occur if a component of \( x(t) \) has large amplitude compared to the vector norm of \( b \). For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy the building.

Let \( \text{Max}(c) \) denote the maximum modulus of the components of vector \( c \).

Plot \( g(T) = \text{Max}(c(w)) \) with \( w = (2 \cdot \text{Pi})/T \) for periods \( T = 1 \) to \( T = 5 \), ordinates \( \text{Max} = 0 \) to \( \text{Max} = 10 \), the vector \( c(w) \) being the answer produced in L8.3 above. Compare your figure to the textbook Figure 5.3.18.
Sample maple code to define the function Max(c), 4-floor building.

With(linalg):

w:='w': Max:= c -> norm(c,infinity); u:=w*w:
b:=0.25*w*w*vector([1,1,1,1]):
A:=matrix([[-20,10,0,0],[10,-20,10,0],[0,10,-20,10],[0,0,10,-10]]):
Au:=evalm(A+u*diag(1,1,1,1)):
C:=ww -> subs(w=ww,linsolve(Au,-b)):
plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);

PROBLEM L8-4. WARNING: Save your file often!!

Define b
Define A
Define Au
Define C
plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);

PROBLEM L8-5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of L8-4 can be used to detect the of earthquake damage for a given ground vibration of period T. A ground vibration (1/4)cos(wt), T=2*Pi/w, will be assumed, as in L8-4.

(a) Replot the amplitudes in L8-4 for periods 1.5 to 5.5 and amplitudes 5 to 10. There will be several spikes.
(b) Create several zoom-in plots, one for each spike, choosing a T-interval that shows the full spike.
(c) Determine from the several zoom-in plots approximate intervals for the period T such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet.

Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet, periods 1.97 to 2.01. This example for the 4-floor problem.

with(linalg): w:='w': Max:= c -> norm(c,infinity); u:=w*w:
Au:=matrix([[-20+u,10,0,0],[10,-20+u,10,0],[0,10,-20+u,10],[0,0,10,-10+u]]):
b:=0.25*w*w*vector([1,1,1,1]):
C:=ww -> subs(w=ww,linsolve(Au,-b)):
plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);

PROBLEM L8-5. WARNING: Save your file often!!

(a) Re-plot the five spikes.
(b) Plot five zoom-in graphs.
(c) Print period ranges.

PeriodRanges:=[one,two,three,four,five];