Math 2280 Extra Credit Problems
Chapter 6
S2015

Submitted work. Please submit one stapled package per chapter. Kindly label problems Extra Credit. Label each problem with its corresponding problem number, e.g., Xc6.1-4. Please attach this printed sheet to simplify your work.

Problem Xc6.1-4. (Phase Portraits)
Find the equilibrium points for the system. Plot a phase diagram using the maple code below.

\[
\begin{align*}
\frac{dx}{dt} &= x - 2y + 3, \\
\frac{dy}{dt} &= x - y + 2.
\end{align*}
\]

Example: Plot the phase diagram of \( u' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} u + \begin{pmatrix} 4 \\ 5 \end{pmatrix} \) using maple.

```
with(DEtools):
equilEQ:=[0=x+2*y+4,0=3*y+5];
solve(equilEQ,{x,y});# find diagram center (a,b)
a:=-2/3;b:=-5/3;
de:=[diff(x(t),t)=x(t)+2*y(t)+4,diff(y(t),t)=3*y(t)+5];
ic:=[[x(0)=0,y(0)=-1],[x(0)=-1,y(0)=-1.5],[x(0)=0.5,y(0)=-2],[x(0)=0.5,y(0)=-1.5],[x(0)=-0.7,y(0)=-1.7]];
DEplot(de,[x(t),y(t)],t=-10..10,ic,x=a-2..a+2,y=b-2..b+2,stepsize=0.05);
```

Problem Xc6.1-8. (Equilibrium Points)
Find the equilibrium points for the system. Plot a phase diagram. The graph window should include the three equilibrium points.

\[
\begin{align*}
\frac{dx}{dt} &= x - 2y, \\
\frac{dy}{dt} &= 4x - x^3.
\end{align*}
\]

Problem Xc6.1-18. (Stability)
Determine if the equilibrium point \((0,0)\) is stable, asymptotically stable, or unstable. Identify the equilibrium point as a node, saddle, center or spiral by examination of its computer-generated direction field.

(a) \( x' = y, \quad y' = -x \)
(b) \( x' = y, \quad y' = -5x - 4y \)
(c) \( x = -2x, \quad y' = -2y \)
(d) \( x = y, \quad y' = x \)

Problem Xc6.2-2. (Classification by Eigenvalues)
Compute the eigenvalues of \( A \). Determine stability of equilibrium \((0,0)\) and classify as node (proper/improper), saddle, spiral, center.

(a) \( A = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \)
(b) $A \begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

c) $A \begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$

c) $A \begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$

Problem Xc6.2-12. (Phase Portrait)
Find the equilibrium point (it is unique) and plot by computer a phase diagram.

\[
\begin{align*}
\frac{dx}{dt} &= x + y - 7, \\
\frac{dy}{dt} &= 3x - y - 5.
\end{align*}
\]

Problem Xc6.2-22. (Almost Linear System)
Linearize the system at its equilibria and determine the stability and type of each. Plot a phase diagram by computer to verify the claims made.

\[
\begin{align*}
\frac{dx}{dt} &= 2x - 5y + x^3, \\
\frac{dy}{dt} &= 4x - 6y + y^4.
\end{align*}
\]

Problem Xc6.3-8. (Predator-Prey System)
Linearize the system at equilibrium point (0,0). Verify that the phase diagram of the nonlinear system at (0,0) is a saddle.

\[
\begin{align*}
\frac{dx}{dt} &= x(5 - x - y), \\
\frac{dy}{dt} &= y(-2 + x).
\end{align*}
\]

Problem Xc6.3-9. (Predator-Prey System)
Linearize the system at equilibrium point (5,0). Verify that the phase diagram of the nonlinear system at (5,0) is a saddle.

\[
\begin{align*}
\frac{dx}{dt} &= x(5 - x - y), \\
\frac{dy}{dt} &= y(-2 + x).
\end{align*}
\]

Problem Xc6.3-10. (Predator-Prey System)
Linearize the system at equilibrium point (2,3). Verify that the phase diagram of the nonlinear system at (2,3) is an asymptotically stable spiral.

\[
\begin{align*}
\frac{dx}{dt} &= x(5 - x - y), \\
\frac{dy}{dt} &= y(-2 + x).
\end{align*}
\]

Problem Xc6.4-4. (Almost Linear System)
Linearize at (0,0) and classify the equilibrium point (0,0) of the nonlinear system, using a phase diagram to verify the conclusion.

\[
\begin{align*}
\frac{dx}{dt} &= 2\sin x + \sin y, \\
\frac{dy}{dt} &= \sin x + 2\sin y.
\end{align*}
\]
Problem Xc6.4-8. (Almost Linear System)
Linearize at all equilibria and classify the equilibrium points of the nonlinear system. Use a phase diagram to verify the conclusions.

\[
\begin{align*}
\frac{dx}{dt} &= y, \\
\frac{dy}{dt} &= \sin \pi x - y.
\end{align*}
\]

End of extra credit problems chapter 6.