

**Math 2280 Extra Credit Problems**  
**Chapter 4**  
**S2015**

**Submitted work.** Please submit one stapled package per chapter. Kindly label problems Extra Credit. Label each problem with its corresponding problem number, e.g., Xc4.1-8. Please attach this printed sheet to simplify your work.

**Problem XcL3.1. (Numerical Solutions)**

You may submit this problem only for score increases on numerical solutions 2.4 to 2.6.

Solve symbolically by chapter 1 methods the initial value problem  $y' = 2xy^2$ ,  $y(0) = 1$ . Do an answer check in maple or by hand. Answer:  $y = 1/(1 - x^2)$ . This problem has no numerical work!

**Problem XcL3.2. (Numerical Solutions)**

You may submit this problem only for score increases on numerical solutions 2.4 to 2.6. This problem counts as three (3) problems.

Solve  $y' = 2xy^2$ ,  $y(0) = 1$  numerically for the value of  $y(0.5)$  using (1) Euler's method, (2) Heun's method and (3) the RK4 method. Use step size  $h = 0.1$ . Include computer code and a print of the data. Report the answers in a table for  $x$ -values 0, 0.1, 0.2, 0.3, 0.4, 0.5.

**Problem XcL4.1. (Numerical Solutions)**

You may submit this problem only for score increases on numerical solutions 2.4 to 2.6.

Solve symbolically by chapter 1 methods the initial value problem  $y' = e^{-y}$ ,  $y(0) = 0$ . Do an answer check in maple or by hand. Answer:  $y = \ln(1 + x)$ . This problem has no numerical work!

**Problem XcL4.2. (Numerical Solutions)**

You may submit this problem only for score increases on numerical solutions 2.4 to 2.6. This problem counts as three (3) problems.

Solve  $y' = e^{-y}$ ,  $y(0) = 0$  numerically for the value of  $y(1.0)$  using (1) Euler's method, (2) Heun's method and (3) the RK4 method. Use step size  $h = 0.001$ . Include a computer code appendix in the report, but do not print the data. Report the answers in a table for  $x$ -values 0, 0.2, 0.4, 0.6, 0.8, 1.0. Include the percentage error  $E = 100|\ln(2) - y(1.0)|/|\ln(2)|$  in your report, one error report for each of the three methods.

**Problem Xc4.1-8. (Transform to a first order system)**

Use the position-velocity substitution  $u_1 = x(t)$ ,  $u_2 = x'(t)$ ,  $u_3 = y(t)$ ,  $u_4 = y'(t)$  to transform the system below into vector-matrix form  $\mathbf{u}'(t) = \mathbf{A}\mathbf{u}(t)$ . Do not attempt to solve the system.

$$x'' - 2x' + 5y = 0, \quad y'' + 2y' - 5x = 0.$$

**Problem Xc4.1-20a. (Dynamical systems)**

Prove this result for system

$$(1) \quad \begin{aligned} x' &= ax + by, \\ y' &= cx + dy. \end{aligned}$$

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**Theorem.** Let  $A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$  and define  $\text{trace}(A) = a + d$ . Then  $p_1 = -\text{trace}(A)$ ,  $p_2 = \det(A)$  are the coefficients in the determinant expansion

$$\det(A - rI) = r^2 + p_1r + p_2$$

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and  $x(t)$  and  $y(t)$  in equation (1) are both solutions of the differential equation  $u'' + p_1u' + p_2u = 0$ .

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**Problem xC4.1-20b. (Solve dynamical systems)**

(a) Apply the previous problem to solve

$$\begin{aligned}x' &= 2x - y, \\y' &= x + 2y.\end{aligned}$$

(b) Use first order methods to solve the system

$$\begin{aligned}x' &= 2x - y, \\y' &= \phantom{2x} + 2y.\end{aligned}$$

**Problem Xc4.2-12. (General solution answer check)**

(a) Verify that  $\mathbf{x}_1(t) = e^{3t} \begin{pmatrix} 1 \\ -1 \end{pmatrix}$  and  $\mathbf{x}_2(t) = e^{2t} \begin{pmatrix} 1 \\ -2 \end{pmatrix}$  are solutions of  $\mathbf{x}' = A\mathbf{x}$ , where

$$A = \begin{pmatrix} 4 & 1 \\ -2 & 1 \end{pmatrix}.$$

(b) Apply the Wronskian test  $\det(\mathbf{aug}(\mathbf{x}_1, \mathbf{x}_2)) \neq 0$  to verify that the two solutions are independent.

(c) Display the general solution of  $\mathbf{x}' = A\mathbf{x}$ .

**End of extra credit problems chapter 4.**