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Problem # 19. Section 5.3
Class: Math 2280-002
Time: 4:30 p.m 03/07/2006

If the cars in Figure 5.3.13 weigh 24 and 8 tons, respectivly, and $k = 1500$ lb/ft, show that the cars separate after $\pi/2$ seconds, and that $x_1'(t) = +\frac{1}{2}v_0$ and $x_2'(t) = +\frac{3}{2}v_0$ thereafter. Thus both cars continue in the original direction of motion, but with defferent velocities.

$$x'' = \begin{bmatrix} -c_1 & c_1 \\ c_2 & -c_2 \end{bmatrix} x$$

Proving exer. 16 so I can use its formulas.

$$|A - \lambda I| = (-c_1 - \lambda)(-c_2 - \lambda) - c_1 c_2 =$$

$$= c_1 c_2 + c_2 \lambda + c_1 \lambda + \lambda^2 - c_1 c_2 =$$

$$= \lambda(\lambda + c_1 + c_2) = 0 \quad \lambda = 0, -c_1 - c_2$$

eigenvalues.

eignvector for $\lambda = 0$

$$\lambda = 0 \Rightarrow \begin{bmatrix} -c_1 & c_1 \\ c_2 & -c_2 \end{bmatrix} v = 0 \quad v = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$$\lambda = -c_1 - c_2 \Rightarrow \begin{bmatrix} -c_1 + c_1 + c_2 & c_1 \\ c_2 & -c_2 + c_1 + c_2 \end{bmatrix} v = \begin{bmatrix} c_2 & c_1 \\ c_2 & c_1 \end{bmatrix} v = 0 \quad v = \begin{bmatrix} c_1 \\ -c_2 \end{bmatrix}$$

eigenvector for $\lambda = -c_1 - c_2$

$m_1 = 1500$ slugs. $m_2 = 500$ slugs.

$$c_1 = \frac{k}{m_1} = \frac{1500}{1500} = 1 \qquad c_2 = \frac{k}{m_2} = \frac{1500}{500} = 3$$

Sub. known values to find solution.

$$x'' = \begin{bmatrix} -1 & 1 \\ 3 & -3 \end{bmatrix} x \Rightarrow \lambda = 0, -4 \Rightarrow v_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, v_{-4} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

Initial condition of the problem 19.

$$x_1 = a_1 + b_1 t + a_2 \cos 2t + b_2 \sin 2t$$

$$x_2 = a_1 + b_1 t - 3a_2 \cos 2t - 3b_2 \sin 2t$$

General solution.

$$x_1' = b_1 - 2a_2 \sin 2t + 2b_2 \cos 2t$$

$$x_2' = b_1 + 6a_2 \sin 2t - 6b_2 \cos 2t$$

Difirentiating and then sub. initial values to find a_1, a_2, b_1, b_2

$$\begin{cases} x_1(0) = a_1 + a_2 = 0 \\ x_2(0) = a_1 - 3a_2 = 0 \\ x_1'(0) = b_1 + 2b_2 = v_0 \\ x_2'(0) = b_1 - 6b_2 = 0 \end{cases} \qquad \begin{cases} a_1 = a_2 = 0 \\ b_1 = \frac{3v_0}{4} \\ b_2 = \frac{v_0}{8} \end{cases}$$

Particular solution.

$x_1(t) = \frac{3v_0}{4}t + \frac{v_0}{8}\sin 2t$

$x_2(t) = \frac{3v_0}{4}t - \frac{3v_0}{8}\sin 2t$

$x_1'(t) = \frac{3v_0}{4}\cancel{t} + \frac{v_0}{4}\cos 2t$

$x_2'(t) = \frac{3v_0}{4}\cancel{t} - \frac{3v_0}{4}\cos 2t$

These hold only so long as spring is compressed Refer to pg 32

that is: $x_2(t) - x_1(t) < 0 \Rightarrow$

$\cancel{\frac{3v_0}{4}t} + \frac{3v_0}{8}\sin 2t - \cancel{\frac{3v_0}{4}t} + \frac{v_0}{8}\sin 2t < 0$

$\frac{v_0}{2}\sin 2t < 0 \Rightarrow \sin 2t < 0 \Rightarrow 2t < \pi$ $\underline{t \le \frac{\pi}{2}}$ Time of separati

The velocities after separation will be the same as at the moment of separation another words we substitute ~~t~~ $\frac{\pi}{2}$ (time of separation) for t into x_1' and x_2' to find final velocities:

$x_1' = \frac{3v_0}{4} + \frac{v_0}{4}\cos\pi = \frac{3v_0}{4} - \frac{v_0}{4} = \frac{1v_0}{2}$

$x_2' = \frac{3v_0}{4} - \frac{3v_0}{4}\cos\pi = 2\left(\frac{3v_0}{4}\right) = \frac{3}{2}v_0$

Velocities after separation.

Answer Check: Problem statment.