Michael Lund
2.1-7

Separate variables and use partial fractions to solve

$$\frac{dx}{dt} = 4x(7-x), \quad x(0)=11$$

Sketch the graphs of several solutions, and highlight the indicated particular solution.

$$\frac{dx}{dt} = 4x(7-x)$$

Prepare for quadrature

$$\frac{dx}{4x(7-x)} = dt$$

Heaviside Method

$$\frac{1}{4x(7-x)} = \frac{\frac{1}{7}}{4x} + \frac{\frac{1}{28}}{7-x}$$

$$\int \frac{1}{28x} + \frac{1}{28(7-x)}\,dx = \int dt$$

$$\frac{1}{28}\int \frac{1}{x} + \frac{1}{7-x} = t + C$$

Integrate Both sides

$$\ln|x| - \ln|7-x| = 28t + C$$

$$\left|\frac{x}{x-7}\right| = Ce^{28t}$$

Exponentiate

$x(0)=11$

Use initial conditions

$$\frac{11}{4} = C$$

Solve for $x(t)$

$$\frac{x}{x-7} = \frac{11}{4}e^{28t}$$

$$4x = 11(x-7)e^{28t}$$

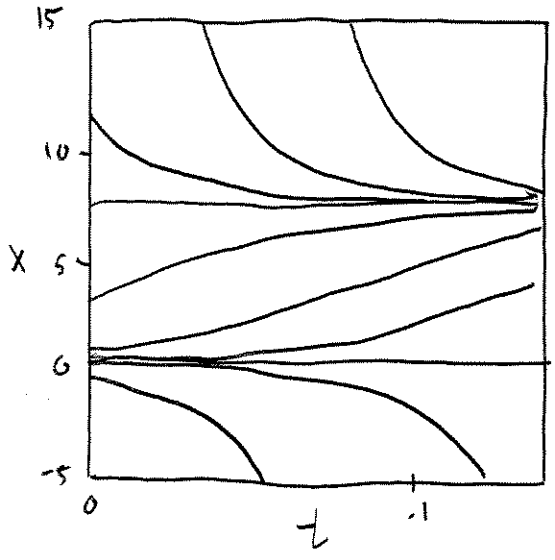
$$= 11e^{28t}x - 77e^{28t}$$

$$4x - 11e^{28t}x = -77e^{28t}$$

$$x(t) = \frac{77e^{28t}}{11 - 11e^{28t}}$$

Checked with Book

Graph:



From Book