Sample Quiz3 Problem 1. A graphic called a phase diagram displays the behavior of all solutions of \( u' = F(u) \). A phase line diagram is an abbreviation for a direction field on the vertical axis (\( u\)-axis). It consists of equilibrium points and signs of \( F(u) \) between equilibria. A phase diagram can be created solely from a phase line diagram, using just three drawing rules:

1. Solutions don’t cross.
2. Equilibrium solutions are horizontal lines \( u = c \). All other solutions are increasing or decreasing.
3. A solution curve can be moved rigidly left or right to create another solution curve.

Use these tools on the equation \( u' = u(u^2 - 4) \) to make a phase line diagram, and then make a phase diagram with at least 8 threaded solutions. Label the equilibria as stable, unstable, funnel, spout, node.

References. Edwards-Penney section 2.2. Course document on Stability.

Sample Quiz3 Problem 2. An autonomous differential equation \( \frac{dy}{dx} = F(x) \) with initial condition \( y(0) = y_0 \) has a formal solution

\[
y(x) = y_0 + \int_0^x F(u)du.
\]

The integral may not be solvable by calculus methods. In this case, the integral is evaluated numerically to compute \( y(x) \) or to plot a graphic. There are three basic numerical methods that apply, the rectangular rule (RECT), the trapezoidal rule (TRAP) and Simpson’s rule (SIMP).

Apply the three methods for \( F(x) = \sin(x^2) \) and \( y_0 = 0 \) using step size \( h = 0.2 \) from \( x = 0 \) to \( x = 1 \). Then fill in the blanks in the following table. Use technology if it saves time. Lastly, compare the four data sets in a plot, using technology.

<table>
<thead>
<tr>
<th>( x )-values</th>
<th>0.0</th>
<th>0.2</th>
<th>0.4</th>
<th>0.6</th>
<th>0.8</th>
<th>1.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )-to 10 digits</td>
<td>0.0</td>
<td>0.0026663619</td>
<td>0.0212943557</td>
<td>0.07133622797</td>
<td>0.1657380596</td>
<td>0.3102683017</td>
</tr>
<tr>
<td>( y )-RECT values</td>
<td>0.0</td>
<td>0.0</td>
<td>0.007997866838</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )-TRAP values</td>
<td>0.0</td>
<td></td>
<td>0.02392968750</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>( y )-SIMP values</td>
<td>0.0</td>
<td></td>
<td>0.02129368017</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

References. Edwards-Penney Sections 2.4, 2.5, 2.6, because methods Euler, Modified Euler and RK4 reduce to RECT, TRAP, SIMP methods when \( f(x, y) \) is independent of \( y \), i.e., an equation \( y' = F(x) \). Course document on numerical solution of \( y' = F(x) \) RECT, TRAP, SIMP methods. Wolfram Alpha at [http://www.wolframalpha.com/](http://www.wolframalpha.com/) can do the RECT rule and graphics with input string

\[
\text{integrate } \sin(x^2) \text{ using left endpoint method with interval width 0.2 from } x=0 \text{ to } x=1
\]