Example 1: Classify as separable (S), quadrature (Q), linear (L) or none (N). (1) $y' = 3(xy)^{1/3}$, (2) $y' = xy^2 + 1$, (3) $y' = x \sin(y)$, (4) $y' = y \sin(x)$, (5) $y' = e^{\ln|x|}$, (6) $y' + xy = x^2y$

Answers: (1) S; (2) N; (3) S; (4) S.L; (5) Q,S,L; (6) L.

Example 2: Check explicit answer $y = (x^{3/2} + c)^2$ for $y' = 3\sqrt{x}\sqrt{y}$ on domain $x \geq 0, y \geq 0$.

Example 3: Check implicit answer $\csc(y) \cot(y) = -x^2/2 + c$ for $y' = x \sin(y)$.

Example 4: Let $f(x, y) = 1 - x^2 + y^2 - x^2y^2$. In relation $f(x, y) = F(x)G(y)$, equations $f(x, 0) = F(x)G(0), f(0, y) = F(0)G(y)$ can determine $F, G$ . Explain. Then find one pair $F, G$.

Example 5: Solve using the constant equation shortcut or the quadrature shortcut.
(1) $y' + 2y = 0$, (2) $2y' + 5y = 3$, (3) $2y' = 3$, (4) $3y' = 5y + \pi$.

Example 6: Solve using the integrating factor shortcut for homogeneous equations.
(1) $y' + 8xy = 0$, (2) $2y' + \sin(x)y = 0$, (3) $xy' + \ln|x|y = 0$.

Example 7: Solve a non-separable equation using the integrating factor method.
(1) $xy' + 2y = x^2$, (2) $xy' + 2y = x$, (3) $xy' + 2y \ln|x| = \ln|x|e^{(\ln|x|)^2}$.

Answers: (1) $y = x^2/4 + c/x^2$, (2) $y = x/3 + c/x^2$, (3) $y = \frac{1}{4} e^{(\ln|x|)^2} + c/e^{(\ln|x|)^2}$.

Example 8: Solve the brine tank model $\frac{dx}{dt} = 1/4 - x/16, x(0) = 20$.

Example 9: Solve the brine tank cascade $x' = -x/2, y' = x/2 - y/4, z' = y/4 - z/6$ with $x(0) = 1, y(0) = -2, z(0) = 1.5$. Answer: $x = e^{-t/2}, y = -2e^{-t/2}, z = 1.5e^{-t/2}$

Example 10: Find all equilibrium solutions for $(x^2 + 1)y' = x + 1 - xy^2 - y^2$

Example 11: Solve $y' = (1 - y)y$ by the substitution $u = y/(1 - y)$.

Example 12: Solve $y' = (1 - y)y$ by partial fraction methods. Check the answer from $P' = (a - bP)P$ and the Verhulst formula $P = \frac{aP_0}{bP_0 + (a - bP_0)e^{-at}}$.

Example 13: Assume US population data 5.308, 23.192, 76.212 million for years 100, 1850, 1900, respectively. Find $a, b$ in the Verhulst model $P' = (a - bP)P$. Answer: $a = 0.3155090164, b = 0.00167716$.

Example 14: Solve $y' = 7y(y - 13), y(0) = 17$. See 2.1-8.

Example 15: Draw a phase line diagram for $y' = y(1 - y)^2(y + 1)$. See Section 2.2.

Example 16: Draw a phase diagram for $y' = y^2(y^2 - 4)$. See 2.2-17.

Example 17: Justify why the direction field along a line $x = x_0$ is the same as the direction field along $x = 0$, for any autonomous equation $y' = F(y)$. 

Week 2 Examples