Math 2250
Maple Lab 8: Earthquake project
S2015

Name _____________________________________ Class Time __________

Project 8. Solve problems L8-1 to L8-5. The problem headers:

------ PROBLEM L8.1. EARTHQUAKE MODEL FOR A BUILDING.
------ PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
------ PROBLEM L8.3. UNDETERMINED COEFFICIENTS STEADY-STATE SOL
------ PROBLEM L8.4. PRACTICAL RESONANCE.
------ PROBLEM L8.5. EARTHQUAKE DAMAGE.

FIVE FLOOR Model.
Refer to the textbook of Edwards-Penney, section 7.4 Application (after the section 7.4 exercises).
Consider a building with five floors each weighing 50 tons. Each floor corresponds to a restoring Hooke's force with constant k=5 tons/foot.
Assume that ground vibrations from the earthquake are modeled by (1/4)cos(wt) with period T=2*Pi/w.

PROBLEM L8-1. BUILDING MODEL FOR AN EARTHQUAKE.
Model the 5-floor problem in Maple.
Define the 5 by 5 mass matrix M and Hooke's matrix K for this system and convert Mx''=Kx into the system x''=Ax where A is defined by textbook equation (1), section 7.4 Application.
Sanity check: Mass m=3125, and the 5x5 matrix contains fraction 16/5.

Then find the eigenvalues of the matrix A to six digits, using the Maple command "linalg[eigenvals](A)."
Sanity check: All six eigenvalues should be negative.

# Sample Maple code for a model with 4 floors.
# Use maple help to learn about evalf and eigenvals.
# A:=matrix( [ [-20,10,0,0], [10,-20,10,0],
# [0,10,-20,10], [0,0,10,-10] ] );
# with(linalg): evalf(eigenvals(A));

# Problem L8.1
# Define k, m and the 5x5 matrix A.
# with(linalg): evalf(eigenvals(A));

PROBLEM L8-2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
Refer to figure 7.4.17 in Edwards-Penney.
Find the natural angular frequencies omega=sqrt(-lambda) for the five story building and also the corresponding periods 2Pi/omega, accurate to six digits. Display the answers in a table.
Compare with answers in Figure 7.4.17 (actually a table), for the 7-story case.

# Sample code for a 4x3 table, 4-story building.
# Use maple help to learn about nops and printf.
# ev:=[-10,-1.206147582,-35.32088886,-23.47296354]: n:=nops(ev):
Consider the forced equation $x' = Ax + \cos(wt)b$ where $b$ is a constant vector. The earthquake's ground vibration is accounted for by the extra term $\cos(wt)b$, which has period $T=2\pi/w$. The solution $x(t)$ is the 5-vector of excursions from equilibrium of the corresponding 5 floors. Sought here is not the general solution, which certainly contains transient terms, but rather the steady-state periodic solution, which is known from the theory to have the form $x(t) = \cos(wt)c$ for some vector $c$ that depends only on $A$ and $b$.

Define $b := 0.25 \cdot w^2 \cdot \text{vector}([1,1,1,1,1])$: in Maple and find the vector $c$ in the undetermined coefficients solution $x(t) = \cos(wt)c$. Vector $c$ depends on $w$. As outlined in the textbook, vector $c$ can be found by solving the linear algebra problem $-w^2 c = Ac + b$; see equation (32), section 7.4. Don't print $c$, as it is too complex; instead, print $c[1]$ as an illustration.

# Sample code for defining $b$ and $A$, then solve for $c$, 4-floor case.
# See maple help to learn about vector and linsolve.
# w := 'w': u := w^2: b := 0.25 * u * vector([1,1,1,1]):
# A := matrix([-20,10,0,0], [10,-20,10,0],
# [0,10,-20,10], [0,0,10,-10]):
# Au := evalm(A + u * diag(1,1,1,1));
# c := linsolve(Au, -b):
# evalf(c[1], 2);

PROBLEM L8-3
# Define $w$, $u$, $b$, $A$, $Au$, $c$
# evalf(c[1], 2);

PROBLEM L8-4. PRACTICAL RESONANCE.
Consider the forced equation $x' = Ax + \cos(wt)b$ of L8-3 above with $b := 0.25 \cdot w^2 \cdot \text{vector}([1,1,1,1,1])$. Practical resonance can occur if a component of $x(t)$ has large amplitude compared to the vector norm of $b$. For example, an earthquake might cause a small 3-inch excursion on level ground, but the building's floors might have 50-inch excursions, enough to destroy the building.

Let $\text{Max}(c)$ denote the maximum modulus of the components of vector $c$. Plot $g(T) = \text{Max}(c(w))$ with $w = (2 \cdot \pi) / T$ for periods $T=1$ to $T=5$, ordinates Max=0 to Max=10, the vector $c(w)$ being the answer produced in L8.3 above. Compare your figure to the textbook Figure 7.4.18.
# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, vector, subs and linsolve.
# with(linalg):
# w:='w': Max:= c -> norm(c,infinity); u:=w*w:
# b:=0.25*w*w*vector([1,1,1,1]):
# A:=matrix([ [-20,10,0,0], [10,-20,10,0], [0,10,-20,10], [0,0,10,-10] ]);:
# Au:=evalm(A+u*diag(1,1,1,1));
# C:=ww -> subs(w=ww,linsolve(Au,-b)):
# plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);

# PROBLEM L8.4. WARNING: Save your file often!!!
# w:='w': Max:= c -> norm(c,infinity); u:=w*w:
# Define b
# Define A
# Define Au
# Define C
# plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);

PROBLEM L8-5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of L8-4 can be used to detect the of
earthquake damage for a given ground vibration of period T. A ground
vibration (1/4)cos(wt), T=2*Pi/w, will be assumed, as in L8-4.

(a) Replot the amplitudes in L8-4 for periods 1.5 to 5.5 and amplitudes
5 to 10. There will be several spikes.
(b) Create several zoom-in plots, one for each spike, choosing a
T-interval that shows the full spike.
(c) Determine from the several zoom-in plots approximate intervals for
the period T such that some floor in the building will undergo
excursions from equilibrium in excess of 5 feet.

# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet,
# periods 1.97 to 2.01. This example for the 4-floor problem.
# with(linalg): w:='w': Max:= c -> norm(c,infinity); u:=w*w:
# Au:=matrix([[[-20+u,10,0,0],[10,-20+u,10,0],[0,10,-20+u,10],[0,0,10,-10+u]]]);
# b:=0.25*w*w*vector([1,1,1,1]):
# C:=ww -> subs(w=ww,linsolve(Au,-b)):
# plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);

# PROBLEM L8-5. WARNING: Save your file often!!
#(a) Re-plot the five spikes.
# plot(Max(C(2*Pi/r)),r=1.5..5.5,5..10,numpoints=150);
#(b) Plot five zoom-in graphs.
# one:=[1.79..1.83]:plot(Max(C(2*Pi/r)),r=one,5..10,numpoints=150);
# two:=[??]:plot(Max(C(2*Pi/r)),r=two,5..10,numpoints=150);
# three:=[??]:plot(Max(C(2*Pi/r)),r=three,5..10,numpoints=150);
# four:=[??]:plot(Max(C(2*Pi/r)),r=four,5..10,numpoints=150);
# five:=[??]:plot(Max(C(2*Pi/r)),r=five,5..10,numpoints=150);
#(c) Print period ranges.
# PeriodRanges:=[one,two,three,four,five]