Math 2250 Extra Credit Problems
Chapter 1
S2015

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the paper [Extra Credit]. Label each solved problem with its corresponding problem number, e.g., [Xc10.3-20].

Problem Xc1.2-4. (Quadrature)
Solve \( y' = x^{-2} + x^{-1}, \ y(1) = 2. \)

Problem Xc1.2-10. (Quadrature)
Solve \( y' = xe^{-2x} + x^2, \ y(0) = 2. \)

Problem Xc1.3-8. (Picard’s theorem)
Find a box with center \( x = 0, \ y = 0 \) to which Picard’s theorem applies, verifying also continuity of \( f(x, y) \) and \( f_y(x, y) \) in the box, for the equation
\[
y' = x\sqrt{x + y + 1}, \quad y(0) = 0.
\]

Problem Xc1.3-14. (Peano’s theorem)
Does Peano’s theorem apply to establish existence of at least one solution, for the problem below? Please carefully check the hypothesis of the theorem, which is continuity of \( f(x, y) \) on a box with center \( x = 0, \ y = 1. \)
\[
y' = 3(y - 1)^{1/3}, \quad y(0) = 1.
\]

Problem Xc1.4-6. (Separable DE)
Solve for equilibrium and non-equilibrium solutions (find the general solution).
\[
y' = 2x \sec y.
\]

Problem Xc1.4-17. (Separability test)
Use the test to verify that the equation \( y' = e^x + e^y \) is not separable.

Problem Xc1.4-18. (Separability test)
Find a factorization \( f(x, y) = F(x)G(y) \) for the problem below and then determine all non-equilibrium solutions.
\[
y' = x^2(y^2 + y) + y^2 + x^2y + 2y + x^2 + 1.
\]

Problem Xc1.4-49. (Newton cooling)
A roast is put into an oven whose temperature is 400°F. The meat thermometer was initially at 40°F and after 30 minutes it rose to 90°F. The roast is done when the thermometer reaches 340°F. How long does it take to cook the roast?

Problem Xc1.5-4. (Linear DE)
Solve \( y' - 2xy = e^{x^2}. \)

Problem Xc1.5-16. (Linear DE)
Solve \( y' = (1 - y) \cos x, \ y(\pi) = 2. \)
Problem Xc1.5-24. (Linear DE)
Solve \((x^2 + 4)y' + 3xy = x, \ y(0) = 1\).

Problem Xc1.5-38. (Brine tank)
Solve the brine tank problem
\[
\begin{align*}
x'(t) &= -4x(t), \\
y'(t) &= 4x(t) - 5y(t).
\end{align*}
\]

Problem Xc-Cooling. (Newton cooling)
Consider the linear differential equation \(u' + ku = ka(t), \ u(0) = u_0\), where \(a(t) = 1 + \sin(\pi(t - 3)/12)\). Solve the equation for \(u(t)\) and check your answer using a computer algebra system. Use technology to do the integration.

Problem Xc-SteadyState. (Steady-state periodic solution)
Consider the linear differential equation \(u' + ku = ka(t), \ u(0) = u_0\), where \(a(t) = 1 + \sin(\pi(t - 3)/12)\). Find the steady-state periodic solution of this equation and check your answer in a computer algebra system.

End of extra credit problems chapter 1.