Theory of Equations

The main topics apply to root-finding and factorization of any polynomial, e.g., solve $x^3 + 3x^2 + 2x = 0$ for $x = 0, -1, -2$ and then factor the polynomial into linear factors $x, (x + 1), (x + 2)$.

Quadratic $ax^2 + bx + c = 0$, [Quadratic equation](http://en.wikipedia.org/wiki/Quadratic_equation)

Long Division Algorithm, [Polynomial long division](http://en.wikipedia.org/wiki/Polynomial_long_division)

Descartes’ Rule of Signs, [Descartes’ rule of signs](http://en.wikipedia.org/wiki/Descartes’_rule_of_signs)

Factor Theorem and Root Theorem, [Factor theorem](http://en.wikipedia.org/wiki/Factor_theorem)

Sum and Product of the Roots, [Vieta’s formulas](http://en.wikipedia.org/wiki/Vieta’s_formulas)

Rational Root Theorem, [Rational root theorem](http://en.wikipedia.org/wiki/Rational_root_theorem)
The second order polynomial equation

\[ ax^2 + bx + c = 0 \]

can be solved by the following methods, illustrated in detail in the WikiHow link http://www.wikihow.com/Factor-Second-Degree-Polynomials-(Quadratic-Equations)

**Inverse FOIL Method**, e.g., \( x^2 + 3x + 2 = (x + 2)(x + 1) \) based on guessing linear factors \((x + 2), (x + 1)\), then test with FOIL expansion.

**Difference of Squares**, using identity \( A^2 - B^2 = (A - B)(A + B) \).

**Complete the Square**, using identity \( u^2 + Bu = \left( u + \frac{B}{2} \right)^2 - \left( \frac{B}{2} \right)^2 \).

**Quadratic Formula**, \( x = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a} \).
Cubic Equations

Cardano provided formulas in 1545 to solve any cubic equation

\[ ax^3 + bx^2 + cx + d = 0. \]

The formulas can be found at the link https://proofwiki.org/wiki/Cardano’s_Formula, although computer algebra systems provide an easier interface.

A number of simple higher order equations can be solved by using the theory of equations. An illustration:

\[ x^3 - 3x - 2 = 0. \]

First, rational roots are tried from the list \( \pm 1, \pm 2 \) predicted by the **Rational Root Theorem**. The first root found is \( x = -1 \). The **Factor Theorem** implies \( x - (-1) \) is a factor. Then the **Division Algorithm** applies: \[ \frac{x^3 - 3x - 2}{x + 1} = x^2 - x - 2. \] This quadratic is factored via **Inverse FOIL**: \( (x + 1)(x - 2) \). Then the final factorization is

\[ x^3 - 3x - 2 = (x + 1)(x + 1)(x - 2). \]

The roots are \(-1, -1, 2\).