Sample Quiz 4

Sample Quiz4 Problem 1. The velocity of a crossbow arrow fired upward from the ground is given at different times in the following table.

Time t in seconds	Velocity $v(t)$ in ft/sec	Location
0.000	50	Ground
1.413	0	Maximum
2.980	-45	Near Ground Impact

(a) The velocity can be approximated by a quadratic polynomial

$$v(t) = at^2 + bt + c$$

which reproduces the table data. Find three equations for the coefficients a, b, c. Then solve for them to obtain $a \approx 2.238$, $b \approx -38.55$, $c = 50$.

(b) Assume a drag model $v' = -32 - \rho v$. Substitute the polynomial answer of (a) into this differential equation, then substitute $t = 0$ and solve for $\rho \approx 0.131$.

(c) Solve the model $w' = -32 - \rho w$, $w(0) = 50$ with $\rho = 0.131$.

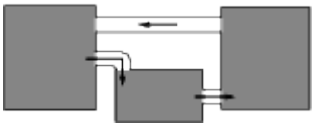
(d) Compare $w(t)$ and $v(t)$ in a plot. Comment on the plot and what it means.

References. Edwards-Penney sections 2.3, 3.1, 3.2. Course documents on Linear algebraic equations and Newton kinematics.

Sample Quiz4 Problem 2. Consider the system of differential equations

$$\begin{aligned} x_1' &= -\tfrac{1}{6}x_1 + \tfrac{1}{6}x_3, \\ x_2' &= \tfrac{1}{6}x_1 - \tfrac{1}{3}x_2, \\ x_3' &= \tfrac{1}{3}x_2 - \tfrac{1}{6}x_3, \end{aligned}$$

for the amounts x_1, x_2, x_3 of salt in recirculating brine tanks, as in the figure:



Recirculating Brine Tanks A, B, C

The volumes are $60, 30, 60$ for A, B, C, respectively.

The steady-state salt amounts in the three tanks are found by formally setting $x_1' = x_2' = x_3' = 0$ and then solving for the symbols x_1, x_2, x_3. Solve the corresponding linear system of algebraic equations to obtain the answer $x_1 = x_3 = 2c$, $x_2 = c$, which means the total amount of salt is uniformly distributed in the tanks in ratio $2 : 1 : 2$.

References. Edwards-Penney sections 3.1, 3.2, 7.3 Figure 5. Course documents on Linear algebraic equations and Systems and Brine Tanks.