Problem 1, part 1.

Panel 1. \[ \frac{dy}{dx} \]

\[ = \frac{d}{dx}(23-18e^{-x}) \]
\[ = 0 + 18e^{-x} \]

RHS = \[ -\frac{q}{h} + 23 \]
\[ = -(23 - 18e^{-x}) + 23 \]
\[ = 18e^{-x} \]

\[ \therefore \text{LHS} = \text{RHS}, \text{ DE } \checkmark \]

Panel 2.

LHS = \[ y(0) \]
\[ = (23 - 18e^{-x})|_{x=0} \]
\[ = 23 - 18e^0 \]
\[ = 5 \]
\[ = \text{RHS}, \text{ IC } \checkmark \]

Problem 1, part 2.

Newton Cooling is \[ y' = -h(y - u), \quad u(0) = u_0 \]. Changing \( y \rightarrow u \) and \( x \rightarrow t \) for the given DE + IC produces
\[
\begin{align*}
    u' &= -(u - 23), \\
    u(0) &= 5,
\end{align*}
\]

The \( h = +1 \) is the cooling constant, \( 23 = u_1 \) = ambient temperature, \( 5 = u_0 \) = initial temperature. Then

\[
\begin{cases}
    y(x) = u(t) = \text{apple temperature}, \\
    23 = u_1 = \text{wall thermometer temp}, \\
    5 = u_0 = \text{apple initial temp}, \\
    -1 = h = \text{Newton Cooling Constant}, \\
    x = t = \text{time}.
\end{cases}
\]
Problem 2, part 1.

The tank could drain any time to < 0 in the past, meaning there is a solution $y(x)$ such that $y(x) > 0$ for $x < t_0$ and $y(x) = 0$ for $x > t_0$. In short, too many solutions. The model fails to determine a unique solution.

Problem 2, part 2.

If $y_0 > 0$, then $f(x, y) = -0.02 \sqrt{y^3}$ and $\frac{\partial f}{\partial y} = -0.01 \sqrt{y^3}$ on box $B = \{(x, y): 1 \leq x \leq 10, \frac{1}{2} y_0 \leq y \leq 10\}$. Picard's Theorem says there is a smaller box $B_1 = \{(x, y): 1 \leq x \leq 10, \frac{1}{2} y_0 \leq y \leq 10\}$ on which a unique edge-to-edge solution $y(x)$ exists, $y(0) = y_0$.

Problem 2, part 3.

The IC is $y(0) = \frac{19}{12}$ feet. Because $y > 0$, then $f(x, y) = F(x)G(y)$ with $F = -0.02$ and $G = \sqrt{y^3}$. Separation gives:

$$\frac{y'}{y^{1/2}} = -0.02$$

$$\int \frac{du}{u^{1/2}} = -0.02 \int dx, \quad u = y^{1/2} \quad \text{method of quadrature}$$

$$\frac{u}{u^{1/2}} = -0.02 x + C_1$$

$$\frac{y}{y^{1/2}} = -0.01 x + C$$

$$y = (-0.01 x + C)^2$$

$$\sqrt{\frac{19}{12}} = (0 + C)$$

$$y = (-0.01 x + \sqrt{\frac{19}{12}})^2$$

Drain time is $x$ when $y = 0$, or $x = \frac{\sqrt{19/12}}{0.01} = 125.83$.

Answer checked in Wolfram Alpha and Waterloo Maple.