Sample Quiz1 Problem 1. An answer check for the differential equation and initial condition

\[ \frac{dy}{dx} = -y(x) + 23, \quad y(0) = 5 \quad (1) \]

requires substitution of the candidate solution \( y(x) = 23 - 18e^{-x} \) into the left side (LHS) and right side (RHS), then compare the expressions for equality for all symbols. The process of testing LHS = RHS applies to both the differential equation and the initial condition, making the answer check have two presentation panels. Complete the following:

1. Show the two panels in an answer check for initial value problem (1).
2. Relate (1) to a Newton cooling model for warming a 5 C apple to room temperature 23 C.


Sample Quiz1 Problem 2. A 2-ft high institutional coffee maker serves coffee from an orifice 5 inches above the base of the cylindrical tank. The tank drains according to the Torricelli model

\[ \frac{dy}{dx} = -0.02\sqrt{|y(x)|}, \quad y(0) = y_0. \quad (2) \]

Symbol \( y(x) \geq 0 \) is the tank coffee height in feet above the orifice at time \( x \) seconds, while \( y_0 \geq 0 \) is the coffee height at time \( x = 0 \).

Establish these facts about the physical problem.

1. If \( y_0 = 0 \), then \( y(x) \) is not determined by the model. A physical explanation is expected, based on possible past tank levels. Numerical solutions are therefore technological nonsense.
2. If \( y_0 > 0 \), then the solution \( y(x) \) is uniquely determined and computable by numerical software. Justify using Picard’s existence-uniqueness theorem.
3. Solve equation (2) using separation of variables when \( y_0 \) is 19 inches, then numerically find the drain time (about 125 seconds).