Quiz 9

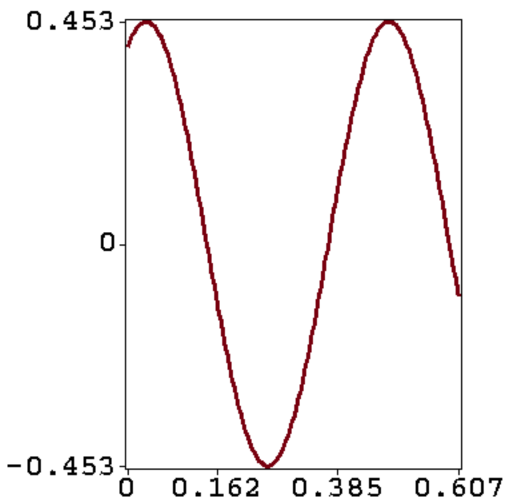
Quiz 9, Problem 1.Harmonic Vibration

A mass of $m = 200$ grams attached to a spring of Hooke's constant k undergoes free undamped vibration. At equilibrium, the spring is stretched 10 cm by a force of 4 Newtons. At time $t = 0$, the spring is stretched 0.4 m and the mass is set in motion with initial velocity 3 m/s directed away from equilibrium. Find:

(a) The numerical value of Hooke's constant k.

(b) The initial value problem for vibration $x(t)$.

(c) Show details for solving the initial value problem for $x(t)$.
The answer is $x(t) = \frac{2}{5}\cos(\sqrt{200}t) + \frac{3}{20}\sqrt{2}\sin(\sqrt{200}t)$, graphed below.



Quiz 9, Problem 2.Harmonic Vibration, Continued

Assume results (a), (b), (c) from Problem 1. In particular, assume

$$x(t) = \frac{2}{5}\cos(\sqrt{200}t) + \frac{3\sqrt{2}}{20}\sin(\sqrt{200}t).$$

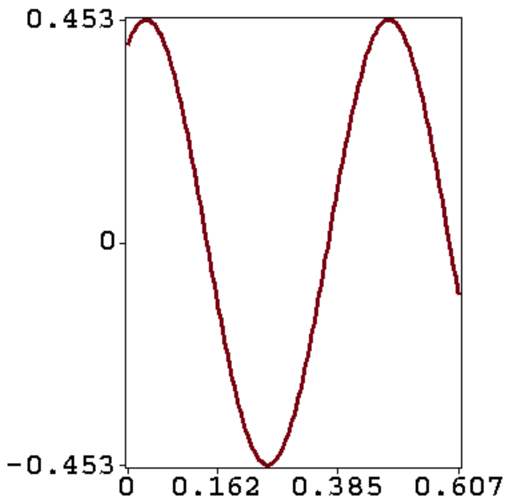
Complete these parts.

(d) Plot the solution $x(t)$ using technology, approximately matching the graphic below.

(e) Show trig details for conversion of $x(t)$ to phase-amplitude form

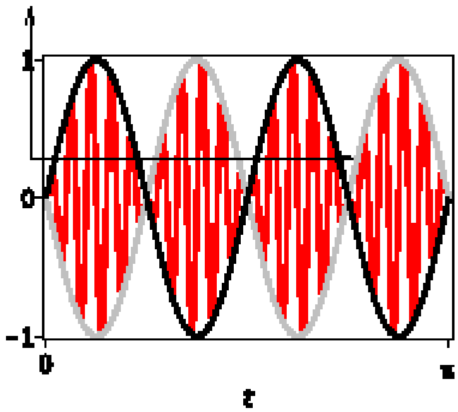
$$x(t) = \frac{\sqrt{82}}{20}\cos(\sqrt{200}t - \arctan(3\sqrt{2}/8)).$$

(f) Report from the answer in part (e) decimal values for the period, amplitude and phase angle. Two-place decimal accuracy is sufficient.



Quiz 9, Problem 3. Beats

The physical phenomenon of **beats** refers to the periodic interference of two sound waves of slightly different frequencies. A destructive interference occurs during a very brief interval, so our impression is that the sound periodically stops, only briefly, and then starts again with a beat, a section of sound that is instantaneously loud again. An illustration of the graphical meaning appears in the figure below.



Beats

Shown in red is a periodic oscillation $x(t) = 2\sin 4t \sin 40t$ with rapidly–varying factor $\sin 40t$ and the two slowly–varying envelope curves $x_1(t) = 2\sin 4t$ (black), $x_2(t) = -2\sin 4t$ (grey).

The undamped, forced spring-mass problem $x'' + 1296x = 640\cos(44t)$, $x(0) = x'(0) = 0$ has by trig identities the solution $x(t) = \cos(36t) - \cos(44t) = 2\sin 4t \sin 40t$.

The Problem. Solve the initial value problem

$$x'' + 1444x = 1056\cos(50t), \quad x(0) = x'(0) = 0$$

by undetermined coefficients and linear algebra, obtaining the solution $x(t) = \cos(38t) - \cos(50t)$. Then show the trig details for $x(t) = 2\sin(6t)\sin(44t)$. Finally, graph $x(t)$ and its slowly varying envelope curves on $0 \le t \le \pi$.