Quiz 1. The problems are due Wednesday, week 2. Please prepare your own report on 8x11 paper, handwritten. You may work in groups and consult Ziwen Zhu and Professor Gustafson for help, office or email, or visit the Math Center in LCB.

Quiz1 Problem 1. An answer check for the differential equation and initial condition

$$\frac{dy}{dx} = -k(y(x) - 73), \quad y(0) = 28 \tag{1}$$

requires substitution of the candidate solution $y(x) = 73 - 45\,e^{-kx}$ into the left side (LHS) and right side (RHS), then compare the expressions for equality for all symbols. The process of testing LHS = RHS applies to both the differential equation and the initial condition, making the answer check have **two** presentation panels. Complete the following:

1. Show the two panels in an answer check for initial value problem (1).

2. Relate (1) to a Newton cooling model for warming a 28 F ice cream bar to room temperature 73 F.

3. Let x be the time in minutes. Find the Newton cooling constant k, given the additional information that the ice cream bar reaches 34 F in 5 minutes.

References. Edwards-Penney sections 1.1, 1.4, 1.5. Newton cooling in Serway and Vuille, *College Physics 9/E*, Brooks-Cole (2011), ISBN-10: 0840062060. Newton cooling differential equation $\frac{du}{dt} = -h(u(t) - u_1)$, Math 2250 slide Three Examples. Math 2250 slide on Answer checks.

Quiz1 Problem 2. A 2-ft high conical water urn drains from an orifice 6 inches above the base. The tank drains according to the Torricelli model

$$|y(x)|^2 \frac{dy}{dx} = -0.021\sqrt{|y(x)|}, \quad y(0) = y_0. \tag{2}$$

Symbol $y(x) \geq 0$ is the tank water height in feet above the orifice at time x seconds, while $y_0 \geq 0$ is the water height at time $x = 0$.

Establish these facts about the physical problem.

1. If $y_0 > 0$, then the solution $y(x)$ is uniquely determined and computable by numerical software. Justify using Picard's existence-uniqueness theorem.

2. Solve equation (2) using separation of variables when y_0 is 18 inches, then numerically find the drain time. Check your answer with technology.

References. Edwards-Penney, Picard's theorem 1 section 1.3 and Torricelli's Law section 1.4. Tank draining `Mathematica` demo at Wolfram Research. Carl Schaschke, *Fluid Mechanics: Worked Examples for Engineers*, The Institution of Chemical Engineers (2005), ISBN-10: 0852954980, Chapter 6. Math 2250 slide on Picard and Peano Theorems. Math 2250 manuscript on applications of first order equations 2250SciEngApplications.pdf, Example 35.