

**Math 2250 Extra Credit Maple Lab 6, Mechanical Oscillations.  
S2015**

**Due date:** See the internet due dates. Maple lab 6 has four problems: L6-1, L6-2, L6-3. Please answer the questions A, B, C, ... associated with each problem.

Problem scores for the Spring 2015 version of maple lab 6, Mechanical Vibrations:

- \_\_\_\_\_ L6-1. [100] Under-Damped Free Vibration.  
 \_\_\_\_\_ L6-2. [100] Undamped Forced Vibration.  
 \_\_\_\_\_ L6-3. [100] Practical Resonance.

### L6-1. Under-Damped Free Vibration

**Free Vibrations.** Consider the problem of free linear vibrations of a damped spring-mass system

$$\begin{aligned} mx'' + cx' + kx &= 0, \\ x(0) &= 0, \quad x'(0) = 1, \\ m &= 4, \quad c = 3. \end{aligned}$$

Symbols  $m$ ,  $c$  and  $k$  are non-negative constants, representing the mass, viscous damping constant and Hooke's constant, respectively. The under-damped case is studied here,  $c^2 < 4km$ , as explained in section 5.4 the Edwards-Penney textbook *Differential Equations and Linear Algebra*, 3/E. Inserting  $m = 4$ ,  $c = 3$  gives the requirement  $9 < 16k$  on Hooke's constant  $k$ .

#### Problem L6-1.

- A. Select and display a positive Hooke's constant  $k$  so that the solution  $x(t)$  is under-damped. Choose the specific value of  $k$  so that the graphic in part B below displays well. Check that  $x(t) = 0$  for infinitely many  $t \geq 0$  (the solution oscillates). Display the exact solution  $x(t)$  obtained by Maple methods as in the example below.
- B. Plot the exact symbolic solution  $x(t)$  on a suitable  $t$ -interval. Check the graphic against Figure 5.4.9 in Edwards-Penney.
- C. Estimate from the graph the decimal value of the pseudo-period. Display the graphical estimate and also the exact pseudo-period  $2\pi/w$ , where  $w$  is the natural frequency of the trigonometric term in the solution  $x(t)$  found above in item L6.1-A.

```
# EXAMPLE(Wrong parameters! Change it!)
# Use semicolons to see what you have done.
# Define the differential equation
de:=3*diff(x(t),t,t)+1.5*diff(x(t),t)+4*x(t)=0:
# Solve the characteristic equation.
solve(3*r^2+1.5*r+4=0,r);
# Define the initial conditions
ic:=x(0)=0,D(x)(0)=1:
# Symbolically solve for x(t)
p:=dsolve({de,ic},x(t),method=laplace):
# Capture the dsolve symbolic solution as a function X(t)
X:=unapply(rhs(p),t):
# Plot the solution
plot(X(t),t=0..5);
```

**Maple tip:** Click with the mouse on the graphic to print the cursor location (left upper corner of the maple window). The coordinates printed are of the form  $(x, y)$ . From this coordinate information, a subtraction estimates the period.

### L6-2. Undamped Forced Vibration

**Forced Linear Vibrations.** Consider the undamped ( $c = 0$ ) forced vibration problem for a spring-mass system:

$$\begin{aligned} mx'' + kx &= 5 \cos(wt), \\ x(0) &= 0, \quad x'(0) = 0, \\ m &= 5, \quad k = 3.5 \end{aligned}$$

Symbol  $w$  is a positive constant, the input natural frequency. Symbols  $m, k$  are respectively mass and Hooke's constant.

- A. Divide the differential equation by  $m = 5$  to obtain  $x'' + w_0^2 x = \cos(wt)$  where  $w_0^2 = k/m = 3.5/5$  defines the natural angular frequency  $w_0 = \sqrt{35/50} = 0.8366600265$ . Choose the input natural frequency  $w$  to be 3 times larger than the natural angular frequency  $w_0$ . Solve for  $x(t)$  using Maple's `dsolve()`.

- B. Because  $w = 3w_0$ , then  $w \neq w_0$ . The solution  $x(t)$  is the sum of two functions, one of period  $2\pi/w$  and the other of period  $2\pi/w_0$  (phenomenon of Beats). Graph the slowly-varying envelope curves and the rapidly-varying solution curve  $x(t)$  on a suitable interval. See Figure 5.6.3 in Edwards-Penney.
- C. Suggest a value for the forcing frequency  $w$  so that the vibration exhibits resonance. Show the resonant behavior in a graphic. Check against Figure 5.6.4 in Edwards-Penney.

### L6-3. Practical Resonance

Consider the damped forced vibration problem

$$\begin{aligned} mx'' + cx' + kx &= 5 \cos(wt), \\ x(0) &= 0, \quad x'(0) = 0, \\ m &= 4, \quad k = 41. \end{aligned}$$

Symbol  $w$  is a positive constant, the input natural frequency. Symbols  $m, c, k$  are respectively mass, viscous damping constant and Hooke's constant.

- A. Consider the damping constants  $c = 2$ ,  $c = 1$  and  $c = 1/2$ . Compute the amplitude function  $C(w)$  [section 5.6] for these three equations, then plot for  $w = 0$  to  $w = 20$  the three amplitude graphs on a single set of axes. Compare against Figure 5.6.9 in Edwards-Penney (it has one curve, yours has 3 curves).
- B. For each case  $c = 2$ ,  $c = 1$ ,  $c = 1/2$ , print the values  $w^*$ ,  $C^*$  where  $C^* = C(w^*) = \max\{C(w) : 0 \leq w \leq 20\}$ . The three data pairs should show that  $C^*$  becomes larger as  $c$  tends to zero.

**Maple Hint:** Use Maple's mouse interface on the graphic of Part A. Specifically, click on a possible maximum (horizontal tangent) in the graph to display the values  $w^*$ ,  $C^*$  on the screen. Look around the screen to see where maple printed the  $x, y$ -coordinates! Copy the values into your maple worksheet report.

```
#EXAMPLE(Beware! Wrong values!)
F:=15: m:=1: k:=25:
c:='c': w:='w':
C:=(w,c)->F/sqrt((k-m*w*w)^2+(c*w)^2):
plot({C(w,4),C(w,3),C(w,2)},w=0..15,color=black);
```