Name ________________________ Class Time _____

Math 2250 Extra Credit Problems
Chapter 9
S2015

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the paper **Extra Credit**. Label each solved problem with its corresponding problem number, e.g., Xc10.3-20.

Problem Xc9.1-4. (Phase Portraits)

Find the equilibrium points for the system. Plot a phase diagram using technology.

$$\begin{cases} \dfrac{dx}{dt} &= x - 2y + 3, \\ \dfrac{dy}{dt} &= x - y + 2. \end{cases}$$

Maple Example: Plot the phase diagram of $\mathbf{u}' = \begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix} \mathbf{u} + \begin{pmatrix} 4 \\ 5 \end{pmatrix}$ using `maple`.

```
with(DEtools):
equilEQ:=[0=x+2*y+4,0=3*y+5];
solve(equilEQ,{x,y});# find diagram center (a,b)
a:=-2/3;b:=-5/3;
de:=[diff(x(t),t)=x(t)+2*y(t)+4,diff(y(t),t)=3*y(t)+5];
ic:=[[x(0)=0,y(0)=-1],[x(0)=-1,y(0)=-1.5],[x(0)=0.5,y(0)=-2],
[x(0)=0.5,y(0)=-1.5],[x(0)=-0.7,y(0)=-1.7]];
DEplot(de,[x(t),y(t)],t=-10..10,ic,x=a-2..a+2,y=b-2..b+2,stepsize=0.05);
```

The plot can also be done in maple version 12+ with the Phase Portrait tool. Start at the TOOLS menu, then TASKS $\longrightarrow$ BROWSE $\longrightarrow$ DIFFERENTIAL EQUATIONS. See the problem notes at the course web site for Chapter 9.

Problem Xc9.1-8. (Equilibrium Points)

Find the equilibrium points for the system. Plot a phase diagram. The graph window should include the three equilibrium points.

$$\begin{cases} \dfrac{dx}{dt} &= x - 2y, \\ \dfrac{dy}{dt} &= 4x - x^3. \end{cases}$$

Problem Xc9.1-18. (Stability)

Determine if the equilibrium point $(0, 0)$ is stable, asymptotically stable, or unstable. Identify the equilibrium point as a node, saddle, center or spiral by examination of its computer-generated direction field.

(a) $x' = y$, $y' = -x$

(b) $x' = y$, $y' = -5x - 4y$

(c) $x' = -2x$, $y' = -2y$

(d) $x' = y$, $y' = x$

Problem Xc9.2-2. (Classification by Eigenvalues)

Compute the eigenvalues of A. Determine stability of equilibrium $(0, 0)$ and classify as node (proper/improper), saddle, spiral, center.

(a) $A\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

(b) $A\begin{pmatrix} 1 & 2 \\ 2 & 1 \end{pmatrix}$

(c) $A\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$

(c) $A\begin{pmatrix} 1 & -2 \\ 2 & -3 \end{pmatrix}$

Problem Xc9.2-12. **(Phase Portrait)**

Find the equilibrium point (it is unique) and plot by computer a phase diagram.

$$\begin{cases} \dfrac{dx}{dt} &= x + y - 7, \\ \dfrac{dy}{dt} &= 3x - y - 5. \end{cases}$$

Problem Xc9.2-22. **(Almost Linear System)**

Linearize the system at its equilibria and determine the stability and type of each. Plot a phase diagram by computer to verify the claims made.

$$\begin{cases} \dfrac{dx}{dt} &= 2x - 5y + x^3, \\ \dfrac{dy}{dt} &= 4x - 6y + y^4. \end{cases}$$

Problem Xc9.3-8. **(Predator-Prey System)**

Linearize the system at equilibrium point $(0,0)$. Verify that the phase diagram of the nonlinear system at $(0,0)$ is a saddle.

$$\begin{cases} \dfrac{dx}{dt} &= x(5 - x - y), \\ \dfrac{dy}{dt} &= y(-2 + x). \end{cases}$$

Problem Xc9.3-9. **(Predator-Prey System)**

Linearize the system at equilibrium point $(5,0)$. Verify that the phase diagram of the nonlinear system at $(5,0)$ is a saddle.

$$\begin{cases} \dfrac{dx}{dt} &= x(5 - x - y), \\ \dfrac{dy}{dt} &= y(-2 + x). \end{cases}$$

Problem Xc9.3-10. **(Predator-Prey System)**

Linearize the system at equilibrium point $(2,3)$. Verify that the phase diagram of the nonlinear system at $(2,3)$ is an asymptotically stable spiral.

$$\begin{cases} \dfrac{dx}{dt} &= x(5 - x - y), \\ \dfrac{dy}{dt} &= y(-2 + x). \end{cases}$$

Problem Xc9.4-4. (Almost Linear System)

Linearize at $(0,0)$ and classify the equilibrium point $(0,0)$ of the nonlinear system, using a phase diagram to verify the conclusion.

$$\left\{\begin{array}{rcl}\dfrac{dx}{dt} & = & 2\sin x + \sin y,\\ \dfrac{dy}{dt} & = & \sin x + 2\sin y.\end{array}\right.$$

Problem Xc9.4-8. (Almost Linear System)

Linearize at all equilibria and classify the equilibrium points of the nonlinear system. Use a phase diagram to verify the conclusions.

$$\left\{\begin{array}{rcl}\dfrac{dx}{dt} & = & y,\\ \dfrac{dy}{dt} & = & \sin \pi x - y.\end{array}\right.$$

End of extra credit problems chapter 9.