Name ____________________________ Class Time _____

Math 2250 Extra Credit Problems
Chapter 4
S2015

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the paper **Extra Credit**. Label each solved problem with its corresponding problem number, e.g., Xc10.3-20.

Problem Xc4.1-20. (Independence)

mapleTest independence or dependence: $\begin{pmatrix} 5 \\ 5 \\ 4 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 2 \end{pmatrix}$.

Problem Xc4.1-30. (Kernel theorem)

Verify from the kernel theorem (Theorem 2, 4.2) that the set of all vectors in $\mathcal{R}^3$ such that $2x - y = 3z$ is a subspace of $\mathcal{R}^3$.

Problem Xc4.1-32. (Subspace criterion)

Apply the subspace criterion (Theorem 1, 4.2) to verify that the set of all vectors in $\mathcal{R}^3$ such that $2x - y = 3z$, $xy = 0$ is **not** a subspace of $\mathcal{R}^3$.

Problem Xc4.2-4. (Subspace criterion)

Apply the subspace criterion (Theorem 1, 4.2) to verify that the set of all vectors in $\mathcal{R}^3$ satisfying $|x| = y + z$ fails to be a subspace of $\mathcal{R}^3$.

Problem Xc4.2-28. (Kernel theorem)

Let S be the subset of $\mathcal{R}^4$ defined by the equations

$$\begin{pmatrix} 1 & 2 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \quad \begin{pmatrix} 4 & 0 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}.$$

Use the kernel theorem (Theorem 2, 4.2) to prove that S is a subspace of $\mathcal{R}^4$.

Problem Xc4.3-18. (Dependence and frame sequences)

Give the vectors below, display a frame sequence from the augmented matrix C of the vectors to final frame $\mathbf{rref}(C)$. Use the sequence to decide if the vectors are independent or dependent. If dependent, then report all possible dependency relations $c_1\mathbf{v}_1 + c_2\mathbf{v}_2 + c_3\mathbf{v}_3 = \mathbf{0}$.

$$\mathbf{v}_1 = \begin{pmatrix} 3 \\ 9 \\ 0 \\ 5 \end{pmatrix}, \quad \mathbf{v}_2 = \begin{pmatrix} -2 \\ 4 \\ -10 \\ 10 \end{pmatrix}, \quad \mathbf{v}_3 = \begin{pmatrix} 4 \\ 7 \\ 5 \\ 0 \end{pmatrix}.$$

Problem Xc4.3-24. (Independence in abstract vector spaces)

Let V be an abstract vector space whose packages of data items $\mathbf{v}$ have unknown details. You are expected to use only definitions and the toolkit of 8 properties in the details of this problem.

Assume given $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ independent vectors in V. Define $\mathbf{u}_1 = \mathbf{v}_1 + 2\mathbf{v}_2$, $\mathbf{u}_2 = \mathbf{v}_3 + \mathbf{u}_1$, $\mathbf{u}_3 = \mathbf{v}_1 + \mathbf{u}_2$. Prove that $\mathbf{u}_1$, $\mathbf{u}_2$, $\mathbf{u}_3$ are independent.

Problem Xc4.4-6. (Basis)

Find a basis for $\mathcal{R}^3$ which includes two independent vectors $\mathbf{v}_1$, $\mathbf{v}_2$ which are in the plane $2x - 3y + 5z = 0$ and a vector $\mathbf{v}_3$ which is perpendicular to both $\mathbf{v}_1$ and $\mathbf{v}_2$. Hint: review the cross product, a topic from calculus.

Problem Xc4.4-24. (Basis for $A\mathbf{x} = \mathbf{0}$)

Display a frame sequence starting with A having final frame $\mathbf{rref}(A)$. Use this sequence to find the scalar general solution of $A\mathbf{x} = \mathbf{0}$ and then the vector general solution of $A\mathbf{x} = \mathbf{0}$. Finally, report a basis for the solution space of $A\mathbf{x} = \mathbf{0}$.

$$A = \begin{pmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 1 & 2 & 3 & 11 \\ 2 & 6 & 4 & 8 \end{pmatrix}, \quad \mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix}.$$

Problem Xc4.5-6. (Row and columns spaces)

Find a basis for the row space and the column space of A, but the bases reported must be rows of A and columns of A, respectively.

$$A = \begin{pmatrix} 1 & -4 & -3 & -7 \\ 2 & -1 & 1 & 7 \\ 2 & 6 & 4 & 8 \end{pmatrix}$$

Problem Xc4.5-24. (Redundant columns)

Use the pivot theorem (Algorithm 2, 4.5) to find the non-pivot columns of A (called redundant columns).

$$A = \begin{pmatrix} 1 & -4 & -3 & -7 & 4 & 3 \\ 2 & -1 & 1 & 7 & 2 & 0 \\ 1 & 2 & 3 & 11 & 0 & 0 \\ 2 & 6 & 4 & 8 & 0 & 1 \end{pmatrix}$$

Problem Xc4.5-28. (Rank and the three properties)

Suppose A is a 5×4 matrix and $A\mathbf{x} = \mathbf{0}$ has a basis of size 3. What are the possible forms of $\mathbf{rref}(A)$?

Problem Xc4.6-2. **(Orthogonality)**

Let A be a 3×3 matrix such that $AA^T = I$. Prove that the columns $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ of A satisfy the orthogonality relations

$$|\mathbf{v}_1| = |\mathbf{v}_2| = \mathbf{v}_3| = 1, \quad \mathbf{v}_1 \cdot \mathbf{v}_2 = \mathbf{v}_2 \cdot \mathbf{v}_3 = \mathbf{v}_3 \cdot \mathbf{v}_1 = 0.$$

Problem Xc4.7-10. **(Subspaces of function spaces)**

Let V be the function space of all polynomials of degree less than 5. Define S to be the subset of V consisting of all polynomials $p(x)$ in V such that $p(0) = p(1)$ and $\int_{-1}^{1} p(x)dx = p(2)$. Prove that S is a subspace of V.

Problem Xc4.7-20. **(Partial fractions and independence)**

Assume the polynomials 1, x, $\ldots$, x^n are *independent*. They are a basis for a vector space V. Use this fact explicitly in the details for determining the constants A, B, C, D in the partial fraction expansion

$$\frac{x^2 - x + 1}{(x-1)(x+1)^2(x-3)} = \frac{A}{x-1} + \frac{B}{x+1} + \frac{C}{(x+1)^2} + \frac{D}{x-3}.$$

Problem Xc4.7-26. **(Solution space basis for a DE)**

Find the general solution of $3y'' + 5y' = 0$, containing symbols c_1, c_2 for the arbitrary constants. Take partial derivatives $\partial y/\partial c_1$, $\partial y/\partial c_2$ to identify two functions of x. Prove that these functions are independent and hence find a basis for the solution space of the differential equation. Suggestion: View $3y'' + 5y' = 0$ as two equations $3v' + 5v = 0$ and $y' = v$. Then use elementary first order differential equation methods.

End of extra credit problems chapter 4.