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**Math 2250 Extra Credit Problems**
**Chapter 2**
**S2015**

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**Submitted work**. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the paper $\boxed{\textbf{Extra Credit}}$. Label each solved problem with its corresponding problem number, e.g., $\boxed{\text{Xc10.3-20}}$.

## Problem Xc-Quadratics. (**Quadratic equations**)

Solve the following quadratic equations by hand and then check the answer using technology. Techniques should include FOIL factoring, complete the square, the quadratic formula, root and factor theorems, leading coefficient reconstruction.
(a) $x^2 + x + 1 = 0$
(b) $8x^2 + 2x + 15 = 0$
(c) $5x^2 - 250x + 3125$

## Problem Xc-Plots. (**Plots using technology**)

Plot the following functions with technology. Print the plots in size $2 \times 2$ inches, approximately.
(a) $f(x) = x^2 - 7x + 5$ on $-1 \le x \le 3$
(b) $f(x) = |x - 1| + |2x + 3|$ on $-5 \le x \le -1$
(c) $f(x) = e^x \sinh(x - 1) + e^{-x} \sinh(2x - 3)$ on $-2 \le x \le 1$
(d) $f(x) = \ln|\sin(2x) + \cos(3x) + 3|$ on $-2\pi \le x \le 3\pi$

## Problem Xc2.1-8. (**Verhulst equation**)

Solve $x'(t) = 4x(t)(7 - x(t))$, $x(0) = 11$ by separation of variables and partial fractions. Sketch the solutions, including equilibria. Check your answer from the textbook answer for 2.1-7.

## Problem Xc2.1-16. (**Population dynamics**)

Assume a population $P(t)$ of alligators with $P(0) = P_0$ satisfies the Verhulst equation $P' = (aP - b)P$ with birth rate $aP_0^2 = 11$ and death rate $bP_0 = 12$ [see 2.1-18 in the textbook]. Find the equilibrium solutions, then sketch them along with a suitable sampling of other possible solutions. Determine the time $t$ at which $P(t) = M/10$, where $M = b/a$ is the threshold population (carrying capacity).

## Problem Xc2.2-10. (**Separation of variables, Verhulst DE**)

Solve $x'(t) = x(t)(3 - x(t))$ by separation of variables and partial fractions. Sketch the solutions, including equilibria. Check your answer from the textbook answer for 2.2-4.

## Problem Xc2.3-10. (**Parachute**)

Assume linear drag model (1) $v' = -32 - 0.15v$ before the chute opens and quadratic drag model (2) $v' = -32 - 1.5v$ after. Suppose the fall from 10000 feet uses (1) for 15 seconds. Find the trip time from 10000 feet to the ground (0 feet).

## Problem Xc2.3-20. (**Terminal speed**)

Assume quadratic drag model $v' = -32 + 0.075v^2$ for a parachutist who pulls the ripcord at 10000 feet at speed 10 feet per second. Find the terminal speed of the parachutist and the trip time to the ground from 10000 feet.

## Problem Xc-ExactSol1. (**Exact solution**)

Solve symbolically by chapter 1 methods the initial value problem $y' = 2xy^2$, $y(0) = 1$. Do an answer check by hand. Answer: $y = 1/(1 - x^2)$. This problem has no numerical work!

## Problem Xc-NumSol1. (**Numerical solution by three methods**)

Solve $y' = 2xy^2$, $y(0) = 1$ numerically for the value of $y(0.5)$ using (1) Euler's method, (2) Heun's method and (3) the RK4 method. Use step size $h = 0.1$. Include computer code and a print of the data. Report the answers in a table for $x$-values $0, 0.1, 0.2, 0.3, 0, 4, 0.5$.

## Problem Xc-ExactSol2. (**Exact solution**)

Solve symbolically by chapter 1 methods the initial value problem $y' = e^{-y}$, $y(0) = 0$. Do an answer check in maple or by hand. Answer: $y = \ln(1 + x)$. This problem has no numerical work!

## Problem Xc-NumSol2. (**Numerical solution by three methods**)

Solve $y' = e^{-y}$, $y(0) = 0$ numerically for the value of $y(1.0)$ using (1) Euler's method, (2) Heun's method and (3) the RK4 method. Use step size $h = 0.001$. Include a computer code appendix in the report, but do not print the data. Report the answers in a table for $x$-values 0, 0.2, 0, 4, 0.6, 0.8, 1.0. Include the percentage error $E = 100|\ln(2) - y(1.0)|/|\ln(2)|$ in your report, one error report for each of the three methods.

**End of extra credit problems chapter 2.**