Name ______________________________ Class Time ______

Math 2250 Extra Credit Problems Chapter 10X
Challenging Laplace Applications S2015

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the paper **Extra Credit**. Label each solved problem with its corresponding problem number, e.g., Xc10.3-20.

Problem Xc10X.1. **(Inverse transform)**

Solve for $f(t)$, given $\mathcal{L}(f(t)) = e^{-2s}\dfrac{s}{(s+1)(s^2+1)}$.

Problem Xc10X.2. **(Inverse transform)**

Solve for $f(t)$, given $\mathcal{L}(f(t)) = \dfrac{d}{ds}\left(e^{-2s}\dfrac{s+1}{s^2(s^2+1)}\right)$.

Problem Xc10X.3. **(Inverse transform)**

Solve for $f(t)$, given $\mathcal{L}(f(t)) = \dfrac{s+4}{s^2(s^2+2s+2)} + \dfrac{e^{-2s}}{s(1-e^{-2s})}$. Hint: Look on the inside cover of Edwards-Penney.

Problem Xc10X.4. **(Resolvent Equation and e^{At})**

Leverrier and Faddeeva derived the following recursions for the coefficients in the expansion

$$(sI - A)^{-1} = \sum_{k=0}^{n-1} \frac{s^{n-k-1}}{\det(sI-A)} A_k.$$

$$\det(sI - A) = s^n - \sum_{k=0}^{n-1} c_k s^k, \quad A_0 = I, \quad A_k = A_{k-1}A - c_{n-k}I.$$

Then

$$e^{At} = \mathcal{L}^{-1}\left((sI-A)^{-1}\right) = \sum_{k=0}^{n-1} \mathcal{L}^{-1}\left(\frac{s^{n-k-1}}{\det(sI-A)}\right) A_k.$$

(a) Write out A_0, A_1 for a 2×2 matrix A, given $\det(sI-A) = s^2 - c_1 s - c_0$.
(b) Write out A_0, A_1, A_2 for a 3×3 matrix A, given $\det(sI-A) = s^3 - c_2 s^2 - c_2 s - c_0$.
(c) Apply the Leverrier-Faddeeva formulas to find the matrix exponential e^{At} for the special case

$$A = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}.$$

(d) Check the answer for e^{At} in a computer algebra system.

Problem Xc10X.5. **(Laplace transform)**

Find $\mathcal{L}(f(t))$, given $f(t) = \dfrac{\sinh(t)}{t}$.

Problem Xc10X.6. **(Laplace transform)**

Find $\mathcal{L}(f(t))$, given $f(t) = \dfrac{d}{dt}(\sinh(t)\cosh(t))$.

Problem Xc10X.7. **(Laplace's method)**

Solve by Laplace's method the initial value problem $tx''(t) - 2x'(t) + tx(t) = 0$, $x(0) = 1$, $x'(0) = 0$. Check the answer in maple.

Problem Xc10X.8. **(Laplace's method)**

Solve by Laplace's method the initial value problem $tx''(t)+2x'(t)+x(t) = \delta(t)+\delta(t-1)+\delta(t-2)$, $x(0) = 0$, $x'(0) = 1$. Check the answer in maple using `dsolve({de,ic},x(t),method=laplace)`.

Problem Xc10X.9. **(Backward table)**

Solve for $f(t)$, given $F(s) = \mathcal{L}(f(t))$.

$$F(s) = \frac{5s^4 + 16s^3 + 2560s^2 + 1600s + 327680}{(s^2 + 100)(s^2 + 256)^2}.$$

Problem Xc10X.10. **(Backward table)**

Solve for $f(t)$, given $F(s) = \mathcal{L}(f(t))$.

$$F(s) = \frac{5s^4 - 24s^3 + 80s^2 - 32s + 256}{(s - 2)^3(s^2 + 16)^2}.$$

End of extra credit problems chapter 10X.