Week 3 Examples

Example 1: Solve: (1) $y'' = -9.8$, (2) $y'' = -0.04y' - 9.8$, both with $y(0) = 0$, $y'(0) = 49$.
Answers: (1) $y = -9.8t^2/2 + 49t$; (2) $y = 7350 - 245t - 7350e^{-t/25}$.

Example 2: Let $w = v\sqrt{\rho/g}$ and $p = \frac{1}{\sqrt{g\rho}}$ to replace Newton's quadratic drag model $v' = -g - \rho v|v|$ by $pw' = -1 - w|w|$. Explain rise model $pw' = -1 - w^2$ and fall model $pw' = -1 + w^2$. See Exercise 2.3-13.

Example 3: Solve $-pw' = -w^2 + 1$ and $-pw' = w^2 + 1$ as separable equations. See the previous example. **Answers**: $w(t) = \tanh(c_1 - t/p)$ and $w(t) = \tan(c_2 - t/p)$

Example 4: Verify rise time 4.6 and fall time 4.8 for Newton's quadratic drag model $v' = -9.8 - 0.0011v|v|$, $v(0) = 49$. Use textbook 2.3 formulas or the previous two examples.

Example 5: Find the point $r = r^*$ of zero acceleration in the Jules Verne equation $r'' = -\frac{Gm_1}{(R_1+r)^2} + \frac{Gm_2}{(R_3-r)^2}$. The answer has symbols. Then calculate $r^* \approx 339,620,820$ meters for the earth-moon problem. Reference:
http://www.math.utah.edu/~gustafso/s2015/2250/julesVerneDE2008.pdf

Example 6: Find the exact solution to $y' = x + y/5$, $y(0) = -3$. Then find $y(5)$. ANSWER: $y = 22\,e^{x/5} - 5x - 25$ by the linear integrating factor method. Then $y(5) = 9.8022002$.

Example 7: Apply Euler's method to $y' = x+y/5$, $y(0) = -3$ with target $x^* = 1$ and step size $h = 0.2$. ANSWERS: Pairs $(0, -3), (0.2, -3.12), (0.4, -3.205), (0.6, -3.253), (0.8, -3.253), (1, -3.234)$.

Example 8: Falling baseball. Given $v' = 32 - 0.16v$, $v(0) = 0$, find Euler's method data points with target $x^* = 10$, step size $h = 1$.

ANSWER: $(0, 0), (1, 32), (2, 59), (3, 81), (4, 100), (5, 116), (6, 130), (7, 141), (8, 150), (9, 158), (10, 165)$.

Example 9: Solve $y' = x + y$, $y(0) = 1$. Then evaluate $y(1)$. ANSWER: $y = 2\,e^x - x - 1$, $y(1) = 3.4365637$.

Example 10: Apply Improved Euler to $y' = x + y$, $y(0) = 1$ with target $x^* = 1.0$ and step size $h = 0.1$. Compare to the Euler method for step sizes h=0.1 and h=0.005. ANSWER: Figure 2.5.4.

Example 11: Apply Improved Euler to $y' = (8 - y)y/3$, $y(0) = 1$ with target $x^* = 5.0$ and various step sizes. ANSWER: Figure 2.5.7 and Figure 2.5.8.

Example 12: Apply RK4 to $y' = x + y$, $y(0) = 1$ with target $x^* = 1.0$ using step size $h = 0.5$. Compare with the exact solution $y = 2\,e^x - x - 1$ at $x = 1$.

ANSWER: The exact value is $y(1) = 3.4365637$. The data pairs are $(0, 1), (0.5, 1.7969), (1.0, 3.4347)$ for step size h=0.1. See Figure 2.6.1 and Figure 2.6.2.

Example 13: Skydiver problem $v' = 9.8 - 0.00016(100v + 10v^2 + v^3)$, $v(0) = 0$. Find the terminal speed 35.578 m/s by roots of equations. Find the speed by RK4 methods, step sizes $h = 0.2$ and $h = 0.1$. Display the results in a table for $t = 0$ to $t = 20$ seconds. ANSWER: Figure 2.6.8.