1.3 E&P

Consider The problem

$$\begin{cases} y' = 3(y-1)^{2/3} \\ y(0) = 1 \end{cases}$$

ⓐ Does The Picard–Lindelöf Theorem apply (E&P 1.3, Thm 1)?

ⓑ Does The Peano Theorem apply?

ⓒ Display an equilibrium solution.

ⓓ Display a non-equilibrium solution.

ⓐ No. Let $f(x,y) = 3(y-1)^{2/3}$. Then f is an elementary function of Calculus I, Therefore continuous everywhere it is defined. But

$$\frac{\partial f}{\partial y} = (3)\left(\frac{2}{3}\right)(y-1)^{-1/3}$$

is discontinuous at $y=1$, hence discontinuous in every box with ~~center~~ $x=0, y=1$. Picard's Thm does not apply.

ⓑ Yes. From ⓐ, f is continuous in every box with center $x=0, y=1$.

ⓒ $y=1$, by inspection.

ⓓ $y = 1 + x^3$

Details:

$$\frac{y'}{(y-1)^{2/3}} = 3$$

$$\int \frac{y'\,dx}{(y-1)^{2/3}} = \int 3\,dx$$

$$\frac{(y-1)^{1/3}}{1/3} = 3x + c_1$$

$$(y-1)^{1/3} = x + c$$

$$y = 1 + (x+c)^3$$

$y=1$ at $x=0$ implies $c=0$

$$y = 1 + x^3$$

ans check:

$LHS = y'$
$= 3x^2$

$RHS = 3(y-1)^{2/3}$
$= 3(x^3)^{2/3}$
$= LHS.$