Background

$\ln e^x = x,\ \ e^{\ln y} = y$	In words, the exponential and the logarithm are inverses. The domains are $-\infty < x < \infty$, $0 < y < \infty$.
$e^0 = 1,\ \ln(1) = 0$	Special values, usually memorized.
$e^{a+b} = e^a e^b$	In words, the exponential of a sum of terms is the product of the exponentials of the terms.
$(e^a)^b = e^{ab}$	Negatives are allowed, e.g., $(e^a)^{-1} = e^{-a}$.
$\left(e^{u(t)}\right)' = u'(t)e^{u(t)}$	The *chain rule* of calculus implies this formula from the identity $(e^x)' = e^x$.
$\ln AB = \ln A + \ln B$	In words, the logarithm of a product of factors is the sum of the logarithms of the factors.
$B\ln(A) = \ln\left(A^B\right)$	Negatives are allowed, e.g., $-\ln A = \ln\dfrac{1}{A}$.
$(\ln\lvert u(t)\rvert)' = \dfrac{u'(t)}{u(t)}$	The identity $(\ln(x))' = 1/x$ implies this general version by the *chain rule*.

PROBLEM 2: $\boxed{\text{pg 17 \#2}}$ Jennifer Laht

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Find a function $y = f(x)$ which satisfies the given differ equation $\frac{dy}{dx} = (x-2)^2$ and initial condition $y(2)=1$.

$y'(x) = (x-2)^2$ given initial equation;

$y'(x)\,dx = (x-2)^2\,dx$ apply the method of quadrature

$\int y'(x)\,dx = \int (x-2)^2\,dx$

$y(x) = \frac{(x-2)^3}{3} + C$

$1 = \frac{(2-2)^3}{3} + C$ use $y(2)=1$

$C=1$

$y(x) = \frac{(x-2)^3}{3} + 1$ candidate solution;

Check:

$LHS = y'(x)$

$= \left[\frac{(x-2)^3}{3} + 1\right]'$

$= (x-2)^2 + 0$

$= RHS$ checks with initial different equation

$LHS = y(2)$

$= \left[\frac{(x-2)^3}{3} + 1\right]\Big|_{x=2}$

$= 0 + 1$

$= RHS$ checks with initial condition $y(2)=1$.

1 Example (Decay Law Derivation) Derive the decay law $\dfrac{dA}{dt} = kA(t)$ from the sentence

> *Radioactive material decays at a rate proportional to the amount present.*

Solution: The sentence is first dissected into English phrases **1** to **4**.

1: *Radioactive material*	The phrase causes the invention of a symbol A for the amount present at time t.
2: *decays at a rate*	It means A undergoes decay. Then A changes. Calculus conventions imply the rate of change is dA/dt.
3: *proportional to*	Literally, it means *equal to a constant multiple of.* Let k be the proportionality constant.
4: *the amount present*	The amount of radioactive material present is $A(t)$.

Solution: *Continued ...*

The four phrases are translated into mathematical notation as follows.

Phrases 1 and 2	Symbol dA/dt.
Phrase 3	Equal sign '=' and a constant k.
Phrase 4	Symbol $A(t)$.

Let $A(t)$ be the amount present at time t. The translation is $\dfrac{dA}{dt} = kA(t)$.