Math 2250
Maple Lab 8: Earthquake project
S2015

Name ____________________________________   Class Time _________

Project 8. Solve problems L8-1 to L8-5. The problem headers:

_______   PROBLEM L8.1. EARHQUAKE MODEL FOR A BUILDING.
_______   PROBLEM L8.2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
_______   PROBLEM L8.3. UNDETERMINED COEFFICIENTS STEADY-STATE SOL
_______   PROBLEM L8.4. PRACTICAL RESONANCE.
_______   PROBLEM L8.5. EARTHQUAKE DAMAGE.

FIVE FLOOR Model.
Refer to the textbook of Edwards-Penney, section 7.4 Application (after the section 7.4 exercises).
Consider a building with five floors each weighing 50 tons. Each floor
corresponds to a restoring Hooke's force with constant k=5 tons/foot.
Assume that ground vibrations from the earthquake are modeled by
(1/4)cos(wt) with period T=2*Pi/w.

PROBLEM L8-1. BUILDING MODEL FOR AN EARTHQUAKE.
Model the 5-floor problem in Maple.
Define the 5 by 5 mass matrix M and Hooke's matrix K for this system
and convert Mx''=Kx into the system x''=Ax where A is defined by
textbook equation (1), section 7.4 Application.
Sanity check: Mass m=3125, and the 5x5 matrix contains fraction 16/5.

Then find the eigenvalues of the matrix A to six digits, using the
Maple command "linalg[eigenvals](A)."
Sanity check: All six eigenvalues should be negative.

```
# Sample Maple code for a model with 4 floors.
# Use maple help to learn about evalf and eigenvals.
# A:=matrix([ [-20,10,0,0], [10,-20,10,0],
[0,10,-20,10],[0,0,10,-10]]);
# with(linalg): evalf(eigenvals(A));
```

```
# Problem L8.1
# Define k, m and the 5x5 matrix A.
# with(linalg): evalf(eigenvals(A));
```

PROBLEM L8-2. TABLE OF NATURAL FREQUENCIES AND PERIODS.
Refer to figure 7.4.17 in Edwards-Penney.

Find the natural angular frequencies omega=sqrt(-lambda) for the
five story building and also the corresponding periods
2PI/omega, accurate to six digits. Display the answers in a table .
Compare with answers in Figure 7.4.17 (actually a table), for the 7-story case.

```
# Sample code for a 4x3 table, 4-story building.
# Use maple help to learn about nops and printf.
# ev:=[-10,-1.206147582,-35.32088886,-23.47296354]: n:=nops(ev):
```

```
# Omega:=lambda -> sqrt(-lambda):
# format:="%10.6f  %10.6f  %10.6f\n":
# seq(printf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),
i=1..n);

 # Problem L8-2
# ev:=[fill this in]: n:=nops(ev):
# Omega:=lambda -> sqrt(-lambda): format:="%10.6f  %10.6f  %10.6f\n":
# seq(printf(format,ev[i],Omega(ev[i]),2*evalf(Pi)/Omega(ev[i])),i=1..n);

PROBLEM  L8-3. UNDETERMINED COEFFICIENTS
              STEADY-STATE PERIODIC SOLUTION.
Consider the forced equation x'=Ax+cos(wt)b where b is a constant
vector. The earthquake's ground vibration is accounted for by the extra
term cos(wt)b, which has period T=2Pi/w. The solution x(t) is the
5-vector of excursions from equilibrium of the corresponding 5 floors.
Sought here is not the general solution, which certainly contains
transient terms, but rather the steady-state periodic solution, which
is known from the theory to have the form x(t)=cos(wt)c for some vector
c that depends only on A and b.

Define  b:=0.25*w*w*vector([1,1,1,1,1]): in Maple and find the vector c
in the undetermined coefficients solution x(t)=cos(wt)c.  Vector c
depends on w. As outlined in the textbook, vector c can be found by
solving the linear algebra problem -w^2 c = Ac + b; see equation (32),
section 7.4. Don't print c, as it is too complex; instead, print c[1]
as an illustration.

# Sample code for defining b and A, then solve for c, 4-floor case.
# See maple help to learn about vector and linsolve.
#  w:='w': u:=w*w: b:=0.25*u*vector([1,1,1,1]):
#  A:=matrix([ [-20,10,0,0], [10,-20,10,0],
#            [0,10,-20,10],[0,0,10,-10]]);
# Au:=evalm(A+u*diag(1,1,1,1));
#  c:=linsolve(Au,-b):
#  evalf(c[1],2);

# PROBLEM L8-3
# Define w, u, b, A, Au, c
# evalf(c[1],2);

PROBLEM L8-4. PRACTICAL RESONANCE.
Consider the forced equation  x'=Ax+cos(wt)b  of L8-3 above with
b:=0.25*w*w*vector([1,1,1,1,1]).
Practical resonance can occur if a component of  x(t)  has large
amplitude compared to the vector norm of b. For example, an earthquake
might cause a small 3-inch excursion on level ground, but the
building's floors might have 50-inch excursions, enough to destroy
the building.

Let Max(c) denote the maximum modulus of the components of vector c.
Plot  g(T)=Max(c(w)) with w=(2*Pi)/T for periods T=1 to T=5, ordinates
Max=0 to Max=10, the vector c(w) being the answer produced in L8.3
above.  Compare your figure to the textbook Figure 7.4.18.
```

```
# Sample maple code to define the function Max(c), 4-floor building.
# Use maple help to learn about norm, vector, subs and linsolve.
#  with(linalg):
#  w:='w': Max:= c -> norm(c,infinity); u:=w*w:
#  b:=0.25*w*w*vector([1,1,1,1]):
# A:=matrix([ [-20,10,0,0], [10,-20,10,0], [0,10,-20,10],
[0,0,10,-10]]);
#  Au:=evalm(A+u*diag(1,1,1,1));
#  C:=ww -> subs(w=ww,linsolve(Au,-b)):
#  plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);

# PROBLEM L8.4. WARNING: Save your file often!!!
# w:='w': Max:= c -> norm(c,infinity): u:=w*w:
# Define b
# Define A
# Define Au
# Define C
# plot(Max(C(2*Pi/r)),r=1..5,0..10,numpoints=150);
```

PROBLEM L8-5. EARTHQUAKE DAMAGE.
The maximum amplitude plot of L8-4 can be used to detect the of earthquake damage for a given ground vibration of period T. A ground vibration (1/4)cos(wt), T=2*Pi/w, will be assumed, as in L8-4.

(a) Replot the amplitudes in L8-4 for periods 1.5 to 5.5 and amplitudes 5 to 10. There will be several spikes.
(b) Create several zoom-in plots, one for each spike, choosing a T-interval that shows the full spike.
(c) Determine from the several zoom-in plots approximate intervals for the period T such that some floor in the building will undergo excursions from equilibrium in excess of 5 feet.

```
# Example: Zoom-in on a spike for amplitudes 5 feet to 10 feet,
# periods 1.97 to 2.01. This example for the 4-floor problem.
# with(linalg): w:='w': Max:= c -> norm(c,infinity); u:=w*w:
# Au:=matrix([[-20+u,10,0,0],[10,-20+u,10,0],[0,10,-20+u,10],[0,0,10,-10+u]]);
# b:=0.25*w*w*vector([1,1,1,1]):
# C:=ww -> subs(w=ww,linsolve(Au,-b)):
# plot(Max(C(2*Pi/r)),r=1.97..2,01,5..10,numpoints=150);

# PROBLEM L8-5. WARNING: Save your file often!!
#(a) Re-plot the five spikes.
# plot(Max(C(2*Pi/r)),r=1.5..5.5,5..10,numpoints=150);
#(b) Plot five zoom-in graphs.
# one:=1.79..1.83:plot(Max(C(2*Pi/r)),r=one,5..10,numpoints=150);
# two:=???:plot(Max(C(2*Pi/r)),r=two,5..10,numpoints=150);
# three:=???:plot(Max(C(2*Pi/r)),r=three,5..10,numpoints=150);
# four:=???:plot(Max(C(2*Pi/r)),r=four,5..10,numpoints=150);
# five:=???:plot(Max(C(2*Pi/r)),r=five,5..10,numpoints=150);
#(c) Print period ranges.
# PeriodRanges:=[one,two,three,four,five];
```