Name _____ Class Time _____

Math 2250 Extra Credit Problems
Chapter 1
S2015

Submitted work. Please submit one stapled package with this sheet on top. Kindly check-mark the problems submitted and label the paper **Extra Credit**. Label each solved problem with its corresponding problem number, e.g., Xc10.3-20.

Problem Xc1.2-4. (Quadrature)

Solve $y' = x^{-2} + x^{-1}$, $y(1) = 2$.

Problem Xc1.2-10. (Quadrature)

Solve $y' = xe^{-2x} + x^2$, $y(0) = 2$.

Problem Xc1.3-8. (Picard's theorem)

Find a box with center $x = 0$, $y = 0$ to which Picard's theorem applies, verifying also continuity of $f(x, y)$ and $f_y(x, y)$ in the box, for the equation

$$y' = x\sqrt{x + y + 1}, \quad y(0) = 0.$$

Problem Xc1.3-14. (Peano's theorem)

Does Peano's theorem apply to establish existence of at least one solution, for the problem below? Please carefully check the hypothesis of the theorem, which is continuity of $f(x, y)$ on a box with center $x = 0$, $y = 1$.

$$y' = 3(y - 1)^{1/3}, \quad y(0) = 1.$$

Problem Xc1.4-6. (Separable DE)

Solve for equilibrium and non-equilibrium solutions (find the general solution).

$$y' = 2x \sec y.$$

Problem Xc1.4-17. (Separability test)

Use the test to verify that the equation $y' = e^x + e^y$ is not separable.

Problem Xc1.4-18. (Separability test)

Find a factorization $f(x, y) = F(x)G(y)$ for the problem below and then determine all non-equilibrium solutions.

$$y' = x^2(y^2 + y) + y^2 + x^2y + 2y + x^2 + 1.$$

Problem Xc1.4-49. (Newton cooling)

A roast is put into an oven whose temperature is 400° F. The meat thermometer was initially at 40° F and after 30 minutes it rose to 90° F. The roast is done when the thermometer reaches 340° F. How long does it take to cook the roast?

Problem Xc1.5-4. (Linear DE)

Solve $y' - 2xy = e^{x^2}$.

Problem Xc1.5-16. (Linear DE)

Solve $y' = (1 - y)\cos x$, $y(\pi) = 2$.

Problem Xc1.5-24. (**Linear DE**)

Solve $(x^2+4)y' + 3xy = x$, $y(0) = 1$.

Problem Xc1.5-38. (**Brine tank**)

Solve the brine tank problem

$$\begin{aligned} x'(t) &= -4x(t), \\ y'(t) &= 4x(t) - 5y(t). \end{aligned}$$

Problem Xc-Cooling. (**Newton cooling**)

Consider the linear differential equation $u' + ku = ka(t)$, $u(0) = u_0$, where $a(t) = 1 + \sin(\pi(t-3)/12)$. Solve the equation for $u(t)$ and check your answer using a computer algebra system. Use technology to do the integration.

Problem Xc-SteadyState. (**Steady-state periodic solution**)

Consider the linear differential equation $u' + ku = ka(t)$, $u(0) = u_0$, where $a(t) = 1 + \sin(\pi(t-3)/12)$. Find the steady-state periodic solution of this equation and check your answer in a computer algebra system.

End of extra credit problems chapter 1.