Math 2250 Lab 7 Name: ______________________________

Due Date: 02/26/2015

1. Consider the three vectors

$$\mathbf{u} = \begin{pmatrix} 1 \\ 0 \\ 2 \end{pmatrix}, \mathbf{v} = \begin{pmatrix} 4 \\ -3 \\ 13 \end{pmatrix}, \mathbf{w} = \begin{pmatrix} 2 \\ -3 \\ 9 \end{pmatrix}$$

   (a) Use a reduced row echelon computation to check that the span of these three vectors is not $\mathbb{R}^3$.

   (b) Use your reduced row echelon form computation to write $\mathbf{w}$ as a linear combination of $\mathbf{u}, \mathbf{v}$. (Hint: what augmented matrix would you have if you were solving $c_1\mathbf{u} + c_2\mathbf{v} = \mathbf{w}$ for c_1, c_2?)

   (c) The span of these three vectors is actually a plane through the origin. Every plane through the origin can be defined by an implicit equation $ax+by+cz = 0$ with some constants a, b, c. Find the constants a, b, c so that all points (x, y, z) whose position vectors $(x, y, z)^T$ are in the span of $\mathbf{u}, \mathbf{v}, \mathbf{w}$ lie in the plane given by $ax+by+cz = 0$.

References: Edwards-Penney Sections 3.3, 4.1.

2. Consider the matrix

$$A = \begin{pmatrix} 1 & 0 & 3 \\ 1 & 1 & 4 \\ 0 & 4 & 4 \end{pmatrix}$$

   (a) Find a basis for the solution space of

   $$A\mathbf{x} = 0$$

   and determine the dimension of the space.

   (b) Find all vectors $\mathbf{b} \in \mathbb{R}^3$ such that the matrix equation

   $$A\mathbf{x} = \mathbf{b}$$

   has a solution $\mathbf{x}$. What is a basis for the space of such vectors $\mathbf{b}$? What is the dimension of this space?

   (c) Show that the set of basis vectors from both (a) and (b) constitute a basis for $\mathbb{R}^3$.

References: Edwards-Penney Sections 3.3 3.6, 4.1.

3. (a) Suppose we have a matrix $A = \begin{pmatrix} 3 & 1 \\ 2 & 2 \end{pmatrix}$ and a square determined by points $(0,0),(0,1),(1,1),(1,0)$. What will the image of the square look like under transformation by the matrix A? i.e.

$$\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = A \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

where $(x_1, x_2)^T$ is a point on the original square and $(y_1, y_2)^T$ is its image after transformation by A. Draw the image of the square after this transformation.

(b) Find the area of the parallelogram in (a). (Hint: think of the parallelogram as sitting inside a larger rectangle $0 \le y_1 \le 4, 0 \le y_2 \le 4$). Compare this area to the determinant of A.

(c) Show, for the general $\mathbf{u}, \mathbf{v}$ in the first quadrant, with $\mathbf{v}$ counterclockwise from $\mathbf{u}$, the area of the parallelgram having the vectors $\mathbf{u}, \mathbf{v}$ as adjacent sides (as in the special case above) always equals $\begin{vmatrix} u_1 & v_1 \\ u_2 & v_2 \end{vmatrix}$.

(d) What happens if you compute $\begin{vmatrix} v_1 & u_1 \\ v_2 & u_2 \end{vmatrix}$ instead?

References: Edwards-Penney Sections 3.6, 4.1.