Math 2250 Lab 4 Name: ________________

Due: 2/12/2015

1. A mathematical model for the rate at which a drug disseminates into the bloodstream is given by

$$\frac{dx(t)}{dt} = r - kx(t),$$

where r and k are positive constants. The function $x(t)$ describes the concentration of the drug in the bloodstream at time t. The steady-state drug concentration is the equilibrium solution r/k. Then the general solution is $x(t) = r/k + c/e^{kt}$. Assume that $r = 0.3$, $k = 0.1$ and the initial drug concentration is zero. Then the model is

$$\begin{aligned} \frac{dx(t)}{dt} &= 0.3 - 0.1x(t), \\ x(0) &= 0, \end{aligned}$$

having particular solution $x(t) = 3 - 3\mathrm{e}^{-\frac{t}{10}}$ and steady-state drug concentration 3.

$$\begin{aligned} \frac{dx(t)}{dt} &= 0.3 - 0.1x(t), \\ x(0) &= 0. \end{aligned}$$

Apply the following methods to approximate the solution $x(t)$ on the time interval $0 \leq t \leq 1$. Do these computations by hand (using a calculator or other technology to do each step, but not using programmed code). You may use technology to check your work before you hand it in. Recommended is programmed code supplied in Canvas.

(a) Euler Method: use step size $h = 0.5$, i.e. 2 time steps.

(b) Improved Euler Method: use step size $h = 0.5$ again.

(c) Runge Kutta Method: use step size $h = 1.0$, i.e. just 1 time step.

(d) Compare these approximations for $x(1)$ with the exact solution at $t = 1$ and comment about the relative accuracy of the three techniques, by computing the relative errors

$$\left| \frac{x_{approx} - x_{exact}}{x_{exact}} \right|$$

in each case. The large step sizes cause all methods to have large errors.

References: Edwards & Penney Section 2.4-2.6

2. A package with a parachute attached is dropped from a helicopter. The wind resistance provided by the parachute is non-linear, modeled by the following differential equation and initial condition. Units of v are ft/sec. It is assumed that the parachute opens when the package is released. The drag force is $F(v) = 0.17v + 0.13v^{1.8}$ and 32 ft/sec^2 is the gravitational constant.

$$\begin{cases} v'(t) = 32 - 0.17v - 0.13v^{1.8} \\ v(0) = 0 \end{cases}$$

(a) What is the terminal velocity in this model? **Hint**: The **slope function** $32 - 0.17v - 0.13v^{1.8}$ is a decreasing function of v for $v > 0$ having value 32 when v=0. The slope function limit at $v = \infty$ is $-\infty$. Therefore, the slope function has exactly one root, which you can find with technology, for example `matlab, maple` or `Wolfram Alpha`.

(b) This is a differential equation which does NOT have an elementary solution. Use modified numerical code (e.g., `maple` sources for Euler, Heun and RK4 or `matlab numericsWN.m`) to estimate the solution values at $t = 2$ and $t = 4$ seconds using methods Euler, Improved Euler and Runge Kutta with step sizes $h = 0.2$ and $h = 0.02$.

- Complete the following two tables.

Table 1	Approx $y(2)$, $h = 0.2$	Approx $y(2)$, $h = 0.02$
Euler		
Modified Euler		
Runge-Kutta		

Table 2	Approx $y(4)$, $h = 0.2$	Approx $y(4)$, $h = 0.02$
Euler		
Modified Euler		
Runge-Kutta		

- Use technology to make a first plot for $h = 0.2$. The single graphic shows three approximate solutions on $0 \le t \le 5$ for $h = 0.2$. Label the three solution curves by hand or graphic assist, using labels `Euler, Modified Euler, Runge-Kutta`.
- Make a second plot by repeating the previous part for $h = 0.02$.
- Write a paragraph discussing the apparent accuracy of the three methods. Discuss step sizes $h = 0.2$, $h = 0.02$ and what should happen to the graphic if the step size is divided by 10 again ($h = 0.002$).

  Remark. Since we don't know the exact solution in this problem, we apply a crude test: Compare the numerical approximations for velocity at $t = 2$ and $t = 4$, for Runge-Kutta with $h = 0.2$ and $h = 0.02$. If these approximations are close, then it's likely that they are also close to the exact solution.

References: Edwards & Penney Section 2.4-2.6, the model is similar to that of section 2.6 Example 3.