

Math 2250 Lab 3
Due Date: 02/05/2015

Name: _____

1. Paratrooper with Linear Drag

A paratrooper bails out of an airplane at altitude 15,000 ft, then falls freely for 30 seconds, then opens a parachute. Assume linear air resistance kv ft/s² with drag coefficient $\rho=0.4$ before the parachute opens and $\rho= 4$ after the parachute opens.

Note: A technology check is expected. The drag coefficient is defined by $\rho = k/m$.

- (a) Write down the ODE for velocity v of the paratrooper before the parachute opens.
- (b) Find the velocity at time 5 seconds and the velocity at parachute open time 30 seconds.
- (c) Find the height when the paratrooper opens the parachute.
- (d) Explain why $w' = -32 - 4w$, $w(0) = v(30) \approx -80$ is the model for the paratrooper's descent after the parachute opens.
- (e) Explain why the total time of descent for the paratrooper is $t = 30 + 1598 = 1628$ s.

Reference: Edwards & Penney Section 2.3. This traditional parachute problem can be modified to be more realistic. Read Meade & Struthers (1999) *Differential Equations in the New Millennium: the Parachute Problem*, Int. J. Engng Ed. 15(6), 417-424. Retrieved on January 29, 2015. See also Meade, D (1998), *ODE Models for the Parachute Problem*, SIAM Review. 40(2). 327-332. Retrieved PDF on January 29, 2015 from Utah Marriot Library online journals, SIAM Review, search string *parachute problem*.

2. The growth of a certain bacteria in a reactor is assumed to be governed by the logistic equation:

$$\frac{dP}{dt} = k \cdot P(M - P)$$

where P is the population in millions and t is the time in days. Symbol M is the carrying capacity of the reactor and k is a constant that depends on the growth rate.

- (a) Suppose that the carrying capacity of the reactor is 10 million bacteria, and that the peak growth rate is 3 million bacteria per day. Determine the constants k and M in the above equation.
- (b) Supposing the bioreactor initially contains 250,000 bacteria, find the number of bacteria in the tank. How long will it take for the population to reach 50% of the carrying capacity?
- (c) Suppose that bacteria are being harvested from the tank continuously. Let h be the rate at which the bacteria are harvested in millions per day. Write down the new differential equation governing the bacteria population. What is the maximum rate of harvesting h that will not cause the population of bacteria to go extinct? (Below this rate there will always be one stable equilibrium point where P is positive).

References: Edwards & Penney Sections 2.1, 2.2.

Verhulst differential equation manuscript

<http://www.math.utah.edu/~gustafso/s2015/2250/2250logistic.pdf>

Phase line diagram, phase portrait, stability, bifurcation manuscript

<http://www.math.utah.edu/~gustafso/s2015/2250/2250phaseline.pdf>