

Math 2250 Lab 2 Name/Unid: _____
Due Date: 01/29/2015

1. (**Warm-up**) Consider the differential equation $\frac{dy}{dx} = ky + b$, where k and b are non-zero constants.
 - (a) Find the general solutions of this differential equation using separable equation method.
 - (b) Find the general solutions of this differential equation using linear first-order equation method.

References: Edwards & Penney Section 1.4, 1.5

2. Consider the set up consisting of two brine tanks where water cascades from tank 1 down into tank 2, and then out of tank 2. (see figure 1.5.5 in your text book). Suppose that when the system is set into motion, the first tank contains 20 gallons of brine, while the tank below it contains 40 gallons of pure water. When the system starts, fresh water is pumped into tank 1 at a constant rate of 2 gal/min. At the same time the brine solution in tank 1 drains into tank 2 at a rate of 2 gal/min, and tank 2 drains at this same rate. Hence the volumes in both tanks stay constant. Assume that the solutions in each tank remain well-mixed, so that although the salt concentrations are changing in time, the concentration of salt leaving each tank equals the average concentration in that tank.
- (a) If the first tank originally contains 24 lbs of salt, solve an initial value problem to find the amount $x(t)$ of salt in tank 1 at time t .
1. Find the DE which describes the dynamics of $x(t)$.
 2. Solve the DE.
- (b) Suppose that $y(t)$ is the amount of salt in tank 2 at time t . Show first that $\frac{dy}{dt} = \frac{x}{4} - \frac{y}{8}$. Then solve for $y(t)$ using the function $x(t)$ found in part (a).

References: Edwards & Penney Section 1.5

3. A 24-year-old woman accepts an engineering position with a starting salary of \$50,000 per year. Her salary $S(t)$ increases exponentially at a continuous rate of 5% a year, so that $S(t) = 50e^{t/20}$ thousand dollars per year, after t years. Meanwhile, 12% of her salary is deposited continuously into a retirement account, which accumulates interest at a continuous annual rate of 4%. Let $A(t)$ be the amount in the retirement account after t years, with $A(0) = 0$ thousands of dollars at the time she begins her new job.
- (a) Recall discrete estimation of continuous rate: suppose there is an independent account $B(t)$ accumulating interest at an annual continuous rate of 4%, then estimate the change in the account balance $\Delta B(t)$ in terms of Δt .
 - (b) Estimate ΔA in terms of Δt , to derive the differential equation satisfied by $A(t)$.
 - (c) Compute $A(46)$, the amount available for her retirement at age 70.

References: Edwards & Penney Section 1.4

4. Match each differential equation with the corresponding vector field. For each slope field, sketch a few likely solution curves.

Differential equations:

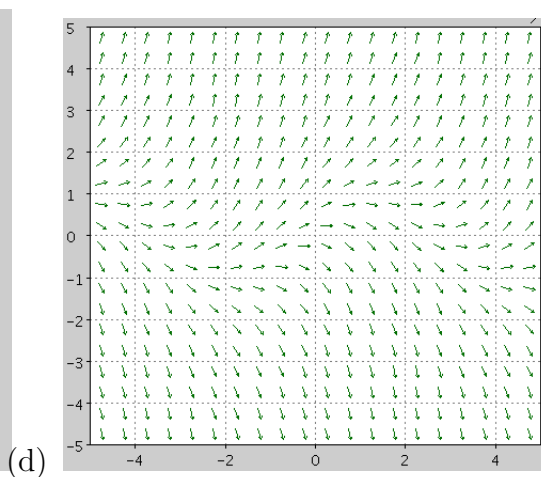
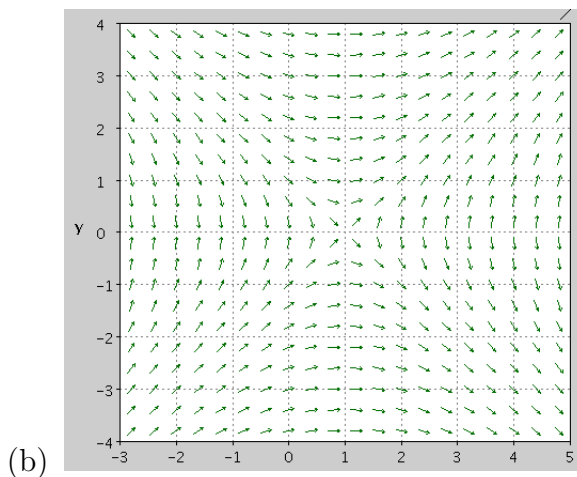
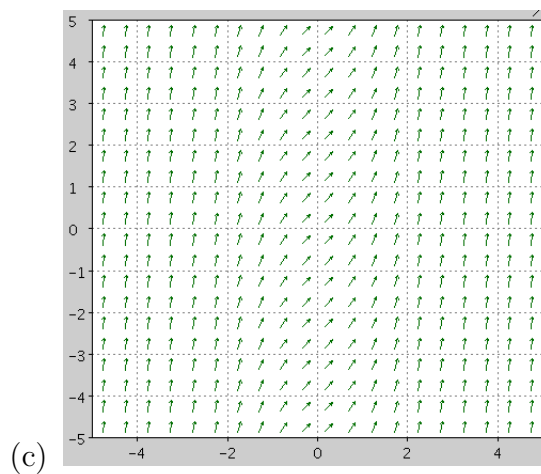
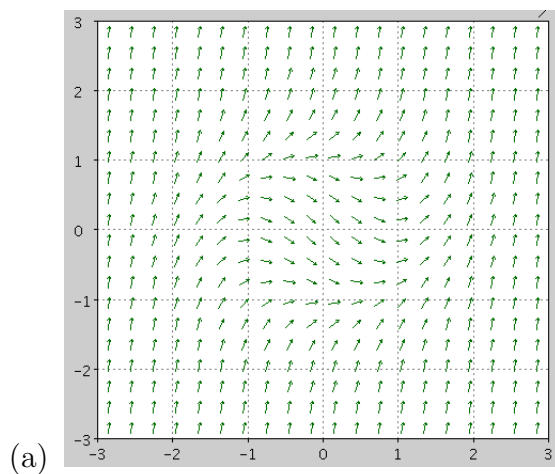
1. $\frac{dy}{dx} = \frac{x-1}{y}$

2. $\frac{dy}{dx} = y - \sin x$

3. $\frac{dy}{dx} = x^2 + y^2 - 1$

4. $\frac{dy}{dx} = x^2 + 1$

Vector fields:



References: Edwards & Penney Section 1.3