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Differential Equations and Linear Algebra 2250

Sample Midterm Exam 2

Version 1, 21 Mar 2014

Instructions: This in-class exam is designed to be completed in 30 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(The 3 Possibilities with Symbols)**
Let a, b and c denote constants and consider the system of equations

$$\begin{pmatrix} 0 & 0 & 0 \\ -2b-4 & 3 & a \\ b+1 & -1 & 0 \\ -1-b & 1 & a \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 0 \\ b^2 \\ b \\ b^2-b \end{pmatrix}$$

(a) [40%] Determine a and b such that the system has a unique solution.

(b) [30%] Explain why $a = 0$ and $b \neq 0$ implies no solution. Ignore any other possible no solution cases.

(c) [30%] Explain why $a = b = 0$ implies infinitely many solutions. Ignore any other possible infinitely many solution cases.

Use this page to start your solution. Attach extra pages as needed.

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2. (Vector Spaces) Do all parts. Details not required for (a)-(d).

(a) [10%] True or false: There is a subspace S of $\mathcal{R}^3$ containing none of the vectors $\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ 2 \end{pmatrix}$.

(b) [10%] True or false: The set of solutions $\vec{u}$ in $\mathcal{R}^3$ of a consistent matrix equation $A\vec{u} = \vec{b}$ can equal all vectors in the xy-plane, that is, all vectors of the form $\vec{u} = (x, y, 0)$.

(c) [10%] True or false: Relations $x^2 + y^2 = 0$, $y + z = 0$ define a subspace in $\mathcal{R}^3$.

(d) [10%] True or false: Equations $x = y$, $z = 2y$ define a subspace in $\mathcal{R}^3$.

(e) [20%] Linear algebra theorems are able to conclude that the set S of all polynomials $f(x) = c_0 + c_1 x + c_2 x^2$ such that $f'(x) + \int_0^1 f(x) x dx = 0$ is a vector space of functions. Explain why $V = \textbf{span}(1, x, x^2)$ is a vector space, then fully state a linear algebra theorem required to show S is a subspace of V. To save time, do not write any subspace proof details.

(f) [40%] Find a basis of vectors for the subspace of $\mathcal{R}^5$ given by the system of restriction equations

$$\begin{array}{rcrcrcrcl} 3x_1 & + & 2x_3 & + & 4x_4 & + & 10x_5 & = & 0, \\ 2x_1 & + & x_3 & + & 2x_4 & + & 4x_5 & = & 0, \\ -2x_1 & & & & & + & 4x_5 & = & 0, \\ 2x_1 & + & 2x_3 & + & 4x_4 & + & 12x_5 & = & 0. \end{array}$$

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3. **(Independence and Dependence)** Do **all parts**.

(a) [10%] State a dependence test for 3 vectors in $\mathcal{R}^4$. Write the hypothesis and conclusion, not just the name of the test.

(b) [10%] State fully an independence test for 3 polynomials. It should apply to show that 1, $1+x$, $x(1+x)$ are independent.

(c) [10%] For any matrix A, **rank**(A) equals the number of lead variables for the problem $A\vec{x} = \vec{0}$. How many non-pivot columns in an 8×8 matrix A with **rank**$(A) = 6$?

(d) [30%] Let $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$, $\mathbf{v}_4$ denote the rows of the matrix

$$A = \begin{pmatrix} 0 & -2 & 0 & -6 & 0 \\ 0 & 2 & 0 & 5 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 3 & 0 \end{pmatrix}.$$

Decide if the four rows $\vec{v}_1$, $\vec{v}_2$, $\vec{v}_3$, $\vec{v}_4$ are independent and display the details of the chosen independence test.

(e) [40%] Extract from the list below a largest set of independent vectors.

$$\vec{v_1} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}, \vec{v_2} = \begin{pmatrix} 0 \\ 2 \\ 2 \\ -2 \\ 0 \\ 2 \end{pmatrix}, \vec{v_3} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \\ 0 \\ 1 \end{pmatrix}, \vec{v_4} = \begin{pmatrix} 0 \\ 3 \\ 3 \\ -1 \\ 0 \\ 5 \end{pmatrix}, \vec{v_5} = \begin{pmatrix} 0 \\ 1 \\ 1 \\ 1 \\ 0 \\ 3 \end{pmatrix}, \vec{v_6} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 2 \\ 0 \\ 2 \end{pmatrix}.$$

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4. **(Determinants)** Do all parts.

(a) [10%] True or False? The value of a determinant is the product of the diagonal elements.

(b) [10%] True or False? The determinant of the negative of the $n \times n$ identity matrix is -1.

(c) [20%] Assume given 3×3 matrices A, B. Suppose $A^2B = E_2E_1A$ and E_1, E_2 are elementary matrices representing respectively a swap and a multiply by -5. Assume $\det(B) = 10$. Let $C = 2A$. Find all possible values of $\det(C)$.

(d) [30%] Determine all values of x for which $(I + C)^{-1}$ fails to exist, where C equals the transpose of the matrix $\begin{pmatrix} 2 & 0 & -1 \\ 3x & 0 & 1 \\ x-1 & x & x \end{pmatrix}$.

(e) [30%] Let symbols a, b, c denote constants. Apply the adjugate [adjoint] formula for the inverse to find the value of the entry in row 3, column 4 of A^{-1}, given A below.

$$A = \begin{pmatrix} 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ a & b & 0 & 1 \\ 1 & c & 1 & 2 \end{pmatrix}$$

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5. (Linear Differential Equations) Do all parts.
(a) [20%] Solve for the general solution of $15y'' + 8y' + y = 0$.
(b) [40%] The characteristic equation is $r^2(2r+1)^3(r^2 - 2r + 10) = 0$. Find the general solution y of the linear homogeneous constant-coefficient differential equation.
(c) [20%] A fourth order linear homogeneous differential equation with constant coefficients has two particular solutions $2e^{3x} + 4x$ and xe^{3x}. Write a formula for the general solution.
(d) [20%] Mark with **X** the functions which **cannot** be a solution of a linear homogeneous differential equation with constant coefficients. Test your choices against this theorem:

The general solution of a linear homogeneous nth order differential equation with constant coefficients is a linear combination of Euler solution atoms.

$e^{\ln\|2x\|}$	e^{x^2}	$2\pi + x$	$\cos(\ln\|x\|)$
$\cos(x \ln\|3.7125\|)$	$x^{-1}e^{-x}\sin(\pi x)$	$\cosh(x)$	$\sin^2(x)$

Use this page to start your solution. Attach extra pages as needed.