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Differential Equations and Linear Algebra 2250

Sample Midterm Exam 1

Version 1, 14 Feb 2014

Instructions: This in-class exam is 50 minutes. No calculators, notes, tables or books. No answer check is expected. Details count 3/4, answers count 1/4.

1. **(Quadrature Equations)**

   (a) [25%] Solve $y' = \dfrac{x^2 - 2}{1 + x}$.

   (b) [25%] Solve $y' = \dfrac{1}{(\cos x + 1)(\cos x - 1)}$.

   (c) [25%] Solve $y' = \dfrac{(1 + e^{2x})^2}{e^x}$, $y(0) = 1$.

   (d) [25%] Find the position $x(t)$ from the velocity model $\dfrac{d}{dt}\left(e^{2t}v(t)\right) = 20e^t$, $v(0) = 0$ and the position model $\dfrac{dx}{dt} = v(t)$, $x(0) = 100$.

Answers supplied after the exam: (a) $y(x) = (1/2)x^2 - x - \ln(1+x) + C$, (b) $y = \cot(x) + C$, (c) $y(x) = -e^{-x} + 2\,e^x + \frac{1}{3}\,e^{3x} - \frac{1}{3}$ (d) $v(t) = 20\,e^{-t} - 20\,e^{-2t}$, $x(t) = -20\,e^{-t} + 10\,e^{-2t} + 110$. The sample exam handwritten solution solved a different problem 1(d), $\dfrac{d}{dt}\left(e^{2t}v(t)\right) = 20e^{-t}$, $v(0) = 0$ and the position model $\dfrac{dx}{dt} = v(t)$, $x(0) = 100$.

Use this page to start your solution. Attach extra pages as needed.

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2. (Classification of Equations)
The differential equation $y' = f(x, y)$ is defined to be **separable** provided functions F and G exist such that $f(x, y) = F(x)G(y)$.

(a) [40%] Check ([X]) the problems that can be converted into separable form. No details expected.

☐ $y' + xy = y^2 + xy^2$	☐ $y' = (x-1)(y+1) + (y-xy)y$
☐ $y' = \cos(xy)$	☐ $e^x y' = x \ln\|y\| + x^2 \ln(y^2)$

(b) [10%] State a partial derivative test that concludes $y' = f(x, y)$ is a linear differential equation but not a quadrature differential equation.

(c) [20%] Apply classification tests to show that $y' = xy^2$ is not a linear differential equation. Supply all details.

(d) [30%] Apply a test to show that $y' = e^x + y \ln|x|$ is not separable. Supply all details.

Use this page to start your solution. Attach extra pages as needed.

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3. (Solve a Separable Equation)
Given $(yx + 2y)y' = ((2 + x)\sin(x)\cos(x) + x)(y^2 + 3y + 2)$.

(a) [80%] Find a non-constant solution in implicit form.
To save time, **do not solve** for y explicitly. No answer check expected.

(b) [20%] Find all constant solutions (also called equilibrium solutions; no answer check expected).

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4. (Linear Equations)

(a) [50%] Solve the linear model $4x'(t) = -160 + \dfrac{24}{2t+3}x(t)$, $x(0) = 30$. Show all integrating factor steps.

(b) [20%] Solve the homogeneous equation $\dfrac{dy}{dx} - (\cos x)y = 0$.

(c) [30%] Solve $11\dfrac{dy}{dx} + 33y = 5$ using the superposition principle $y = y_h + y_p$. Expected are answers for y_h and y_p.

Answers reported after the exam: (a) $x(t) = 20t + 30$, (b) $y(x) = c\,e^{\sin(x)}$, (c) $y(x) = \frac{5}{33} + c\,e^{-3x}$

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5. (Stability)

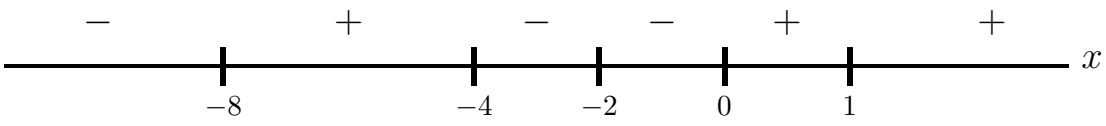
(a) [50%] Draw a phase line diagram for the differential equation

$$\frac{dx}{dt} = \ln(1+x^4)\,(2-|3x-1|)^3\,(2+x)(x^2-4)(1-x^2)^5.$$

Expected in the phase line diagram are equilibrium points and signs of dx/dt.

Answers reported after the exam: Roots $-2, -1, -1/3, 0, 1, 2$. Signs left to right are: MINUS, MINUS, PLUS, MINUS, MINUS, MINUS, PLUS.

(b) [50%] Assume an autonomous equation $x'(t) = f(x(t))$. Draw a phase diagram with at least 12 threaded curves, using the phase line diagram given below. Add these labels as appropriate: **funnel**, **spout**, **node** [neither spout nor funnel], **stable**, **unstable**.



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