

Differential Equations and Linear Algebra

2250-10 7:15am on 6 May 2015

Instructions. The time allowed is 120 minutes. The examination consists of eight problems, one for each of chapters 1-2, 3, 4, 5, 6, 7, 9, 10, each problem with multiple parts. A chapter represents 15 minutes on the final exam.

Each problem on the final exam represents several textbook problems numbered (a), (b), (c), \dots . Each chapter division adds at most 100 towards the maximum final exam score of 800. The final exam grade is reported as a percentage 0 to 100, as follows:

$$\text{Final Exam Grade} = \frac{\text{Sum of scores on eight chapters}}{8}.$$

- Calculators, books, notes, computers and electronics are not allowed.
- Details count. Less than full credit is earned for an answer only, when details were expected. Generally, answers count only 25% towards the problem credit.
- Completely blank pages count 40% or less, at the whim of the grader. More credit is possible if you write something.
- Answer checks are not expected and they are not required. First drafts are expected, not complete presentations.
- Please prepare **exactly one** stapled package of all eight chapters, organized by chapter. Please append scratch work for a chapter immediately following chapter solutions. Any work stapled out of order could be missed, due to multiple graders.
- The graded exams will not be returned. This policy is due to government privacy rules, which eliminate the possibility of a box of graded exams outside my office.
- Records will be posted on **CANVAS**, found from the Registrar's web site link. Recording errors can be reported by email, hopefully as soon as discovered, but also even weeks after grades are posted.

Final Grade. The final exam counts as two midterm exams. For example, if exam scores earned were 90 and 92 and the final exam score is 89, then the exam average for the course is

$$\text{Exam Average} = \frac{90 + 92 + 89 + 89}{4} = 90.$$

Homework, quiz and lab scores are each averages 0–100, weighted respectively 20%, 10%, 10%. The **Coursework Average** = $\frac{1}{4}(\text{HW} + \text{HW} + \text{Quiz} + \text{Lab})$. The course average is computed from the formula

$$\text{Course Average} = \frac{60}{100}(\text{Exam Average}) + \frac{40}{100}(\text{Coursework Average}).$$

Averages posted on CANVAS are internal computations of CANVAS: they are not useful numbers for the above formula. You may not earn more than 100% in any category, regardless of extra credit work. Exam scores are records, unalterable by course work or extra credit.

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Scores

Ch1-2.

Ch3.

Ch4.

Ch5.

Ch6.

Ch7.

Ch9.

Ch10.

Ch1 and Ch2. (First Order Differential Equations)[20%] **Ch1-Ch2(a):**

Find the position $x(t)$ from the velocity model $\frac{d}{dt}(e^t v(t)) = 10e^{2t}$, $v(0) = 0$
 and the position model $\frac{dx}{dt} = v(t)$, $x(0) = 100$.

Answer: in progress

$$v(t) = 20e^{-t} - 20e^{-2t}, \quad x(t) = -20e^{-t} + 10e^{-2t} + 110$$

[10%] **Ch1-Ch2(b):**

Find all equilibrium solutions for $y' = x^3 e^y (2 + \cos(y))(y^2 - 3y + 2)$.

Answer: $y = 2, y = 1$ [20%] **Ch1-Ch2(c):**

Given $y' = \frac{2x^2 + x}{1 + x} \left(\frac{y^4 - 2y^2 + 1}{y} \right)$,

find the non-equilibrium solution in implicit form.

To save time, **do not solve** for y explicitly.

Answer:

The equilibrium solutions are $y = 1$, $y = -1$. The implicit solution is

$$-\frac{1}{2(y^2 - 1)} = x^2 - x + \ln|1 + x| + c$$

[20%] **Ch1-Ch2(d):**

Solve the linear homogeneous equation $2\sqrt{1+x} \frac{dy}{dx} = y$ using the integrating factor shortcut.

Answer: in progress

A constant divided by the integrating factor $W = e^{\int -\cos(x) dx} = e^{-\sin(x)}$.

[10%] **Ch1-Ch2(e):**

Draw a phase line diagram for the differential equation

$$\frac{dx}{dt} = (3 + x)(x^2 - 9)(4 - x^2)^3.$$

Label the equilibrium points, display the signs of dx/dt , and classify each equilibrium point as funnel, spout or node. To save time, **do not draw** a phase portrait.

Answer: in progress

Equilibria $x = -2, -1, 1, 2$. Signs of dx/dt from left to right are **plus, plus, minus, plus, minus**. Node at -2 , Funnel at -1 , Spout at 1 , Funnel at 2 .

[20%] **Ch1-Ch2(f):**

Solve the linear drag model $1000 \frac{dv}{dt} = 50 - 200v$ using superposition $v = v_h + v_p$.

Answer: in progress

Superposition applies: $v = v_h + v_p$ where v_h solves $1000v'(t) + 200v(t) = 0$ and v_p is the equilibrium solution. Then $v_h = \text{constant divided by the integrating factor } W = e^{t/10}$ and $v_p = 5000/100$.
 Answer $v = ce^{-t/10} + 50$.

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Ch3. (Linear Systems and Matrices)[40%] **Ch3(a)**: Consider a 3×5 matrix A and its reduced row echelon form:

$$B = \text{rref}(A) = \begin{pmatrix} 1 & 2 & 0 & 2 & 4 \\ 0 & 0 & 1 & 1 & 1 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}.$$

- (1) Explain in detail why $A\vec{x} = \vec{0}$ and $B\vec{x} = \vec{0}$ have exactly the same solutions.
- (2) Show the linear algebra steps used to find the scalar general solution to the system $B\vec{x} = \vec{0}$.
- (3) Report a basis for the solution space S of $A\vec{x} = \vec{0}$.
- (4) Report the dimension of S .

Answer: in progress[20%] **Ch3(b)**: Define matrix A and vector \vec{b} by the equations

$$A = \begin{pmatrix} -2 & 3 \\ 0 & 4 \end{pmatrix}, \quad \vec{b} = \begin{pmatrix} -3 \\ 1 \end{pmatrix}.$$

For the system $A\vec{x} = \vec{b}$, find x_1, x_2 by Cramer's Rule, showing **all details** (details count 75%).**Answer:** in progress

$$x_1 = \Delta_1/\Delta, \quad x_2 = \Delta_2/\Delta, \quad \Delta = \det \begin{pmatrix} -2 & 3 \\ 0 & -4 \end{pmatrix} = 8, \quad \Delta_1 = \det \begin{pmatrix} -3 & 3 \\ 5 & -4 \end{pmatrix} = -3, \quad \Delta_2 = \det \begin{pmatrix} -2 & -3 \\ 0 & 5 \end{pmatrix} = -10, \quad x_1 = \frac{-3}{8}, \quad x_2 = \frac{-10}{8} = \frac{-5}{4}.$$

[40%] **Ch3(c)**:Determine which values of k correspond to (1) a unique solution, (2) infinitely many solutions and (3) no solution, for the system $A\mathbf{x} = \mathbf{b}$ given by

$$A = \begin{pmatrix} 0 & k-2 & k-3 \\ 1 & 4 & k \\ 1 & 4 & 3 \end{pmatrix}, \quad \mathbf{b} = \begin{pmatrix} -1 \\ 1 \\ k \end{pmatrix}.$$

Answer: in progress

Elimination methods with swap, combo, multiply give $\begin{pmatrix} 1 & 4 & k & 1 \\ 0 & k-2 & 0 & k-2 \\ 0 & 0 & 3-k & k-1 \end{pmatrix}$. Then (1)

Unique solution for $k \neq 2$ and $k \neq 3$; (2) No solution for $k = 3$ [signal equation]; (3) Infinitely many solutions for $k = 2$.

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Ch4. (Vector Spaces)

[30%] **Ch4(a):** Some independence tests below apply to prove that vectors x, x^2, xe^x are independent in the vector space of all continuous functions on $-\infty < x < \infty$. Mark one method and display the details of application (details count 75%).

- ☐ **Wronskian test** Wronskian of functions f, g, h nonzero at $x = x_0$ implies independence of f, g, h .
- ☐ **Atom test** Any finite set of distinct Euler solution atoms is independent.
- ☐ **Sampling test** Let samples a, b, c be given and for functions f, g, h define

$$A = \begin{pmatrix} f(a) & g(a) & h(a) \\ f(b) & g(b) & h(b) \\ f(c) & g(c) & h(c) \end{pmatrix}.$$

Then $\det(A) \neq 0$ implies independence of f, g, h .

Answer: in progress

Details for all tests can be given: let $f(x) = x, g(x) = x^{7/3}, h(x) = e^x$. These are atoms, so the atom test applies. The Wronskian test applies directly, using $x_0 = 1$ to obtain Wronskian determinant value $W \neq 0$. The sampling test applies using samples $a = 1, b = -1, c = 2$ because

then $A = \begin{pmatrix} 1 & 1 & e \\ 1 & 1 & e \\ 2 & 4\sqrt[3]{2} & e^2 \end{pmatrix}$ and $\det(A) \neq 0$.

[20%] **Ch4(b):** Give an example of three vectors $\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3$ for which the nullity of their augmented matrix A is two.

Answer: Let $\mathbf{v}_1 = \mathbf{v}_2 = \mathbf{v}_3$ for any nonzero vector \mathbf{v}_1 .

[20%] **Ch4(c):** Find a 4×4 system of linear equations for the constants a, b, c, d in the partial fraction decomposition of the fraction

$$\frac{3x^2 - 14x + 3}{(x+1)^2(x-2)^2}$$

To save time, **do not solve** for a, b, c, d .

Answer: in progress

[30%] **Ch4(d):**

The 5×6 matrix A below has some independent columns. Report a largest set of independent columns of A , according to the Pivot Theorem.

$$A = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 \\ -3 & 0 & -2 & 0 & 1 & -1 \\ -1 & 0 & 0 & 0 & 1 & 0 \\ 6 & 0 & 6 & 0 & 0 & 3 \\ 2 & 0 & 2 & 0 & 0 & 1 \end{pmatrix}$$

Answer: Find $\text{rref}(A) = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1/2 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$. The pivot columns are 1 and 3.

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Ch5. (Linear Equations of Higher Order)

[10%] **Ch5(a)**: Find a basis for the solution space of $y'' + 4y' + 5y = 0$.

[20%] **Ch5(b)**: Solve for the general solution y of the equation $\frac{d^6 y}{dx^6} + 16\frac{d^4 y}{dx^4} = 0$.

[20%] **Ch5(c)**: Find a basis for the solution space of a linear constant coefficient homogeneous differential equation, given the characteristic equation is $r(r+1)(r^3-r)^2(r^2+2r+5)^2 = 0$.

[20%] **Ch5(d)**: Given $6x''(t) + 2x'(t) + 2x(t) = 5\cos(\omega t)$, which represents a damped forced spring-mass system with $m = 6$, $c = 2$, $k = 2$, answer the following questions.

True ☐ or False ☐. Practical mechanical resonance is at input frequency $\omega = \sqrt{5/2}$.

True ☐ or False ☐. The homogeneous problem is over-damped.

Answer:

(a)

$r^2 + 4r + 5 = 0$, $y = c_1 y_1 + c_2 y_2$, $y_1 = e^{-2x} \cos(x)$, $y_2 = e^{-2x} \sin(x)$.

(b) $r^6 + 16r^4 = 0$, roots $r = 0, 0, 0, 0; 4i, -4i$. Then the Euler atoms are $1, x, x^2, x^3; \cos 4x, \sin 4x$.

The general solution is a linear combination of the atoms.

(c) Write as $r^3(r+1)^3(r-1)^2((r+1)^2+4)^2 = 0$. Then y is a linear combination of the Euler atoms $1, x, x^2, e^{-x}, xe^{-x}, x^2e^{-x}, e^x, xe^x, e^{-x} \cos 2x, xe^{-x} \cos 2x, \sin 2x, xe^{-x} \sin 2x$.

(d) False and False. Use $6r^2 + 2r + 2 = 0$ and the quadratic formula to obtain complex conjugate roots. It is under-damped.

[30%] **Ch5(e)**: Determine for $\frac{d^5 y}{dx^5} + 4\frac{d^3 y}{dx^3} = x + x^2 + e^x + x \cos(2x)$ the shortest trial solution for y_p according to the method of undetermined coefficients. **Do not evaluate** the undetermined coefficients!

Answer:

(e) The homogeneous solution is a linear combination of the atoms $1, x, x^2, \cos 2x, \sin 2x$ because the characteristic polynomial has roots $0, 0, 0, 2i, -2i$.

1 An initial trial solution y is constructed by Rule I for atoms $1, x, e^{3x}, e^{-3x}, \cos x, \sin x$ giving

$$\begin{aligned} y &= y_1 + y_2 + y_3 + y_4, \\ y_1 &= d_1 + d_2 x + d_3 x^2, \\ y_2 &= d_4 e^x, \\ y_3 &= d_5 \cos 2x + d_6 x \cos 2x, \\ y_4 &= d_7 \sin 2x + d_8 x \sin 2x. \end{aligned}$$

Linear combinations of the listed independent atoms are supposed to reproduce, by specialization of constants, all derivatives of the right side of the differential equation.

2 Rule II is applied individually to each of y_1, y_2, y_3, y_4 .

The result is the **shortest trial solution**

$$\begin{aligned} y &= y_1 + y_2 + y_3 + y_4, \\ y_1 &= x^3 d_1 + d_2 x^4 + d_3 x^5, \\ y_2 &= d_4 e^x, \\ y_3 &= d_5 x \cos 2x + d_6 x^2 \cos 2x, \\ y_4 &= d_7 x \sin 2x + d_8 x^2 \sin 2x. \end{aligned}$$

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Ch6. (Eigenvalues and Eigenvectors)

[20%] **Ch6(a)**: Let $A = \begin{pmatrix} -7 & 4 \\ -12 & 7 \end{pmatrix}$. Circle possible eigenpairs of A .

$$\left(1, \begin{pmatrix} 1 \\ 2 \end{pmatrix}\right), \quad \left(2, \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right), \quad \left(-1, \begin{pmatrix} 2 \\ 3 \end{pmatrix}\right).$$

Answer: The first and the last, because the test $A\mathbf{x} = \lambda\mathbf{x}$ passes in both cases.

[20%] **Ch6(b)**: Find the 2×2 matrix A which has eigenpairs

$$\left(0, \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right), \quad \left(2, \begin{pmatrix} 2 \\ 1 \end{pmatrix}\right).$$

Answer: in progress

Write $D = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ and $P = \begin{pmatrix} 1 & 2 \\ 2 & 3 \end{pmatrix}$. Then $CP = PD$ implies $C = PDP^{-1} =$
 $\begin{pmatrix} -7 & 4 \\ -12 & 7 \end{pmatrix}.$

[20%] **Ch6(c)**: Find the eigenvectors corresponding to complex eigenvalues $-1 \pm 3i$ for the 2×2 matrix $A = \begin{pmatrix} -1 & 3 \\ -3 & -1 \end{pmatrix}$.

Answer: $\left(-1 + 3i, \begin{pmatrix} -i \\ 1 \end{pmatrix}\right), \left(-1 - 3i, \begin{pmatrix} i \\ 1 \end{pmatrix}\right)$

[30%] **Ch6(d)**: Let $A = \begin{pmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 0 & -5 \end{pmatrix}$. Display the details for finding all eigenpairs of A .

Answer:

The eigenpairs are

$$\left(-1, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\right), \quad \left(1, \begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix}\right), \quad \left(-5, \begin{pmatrix} 1 \\ 1 \\ -6 \end{pmatrix}\right).$$

An expected detail is the cofactor expansion of $\det(A - \lambda I)$ and factoring to find eigenvalues $-1, 1, -5$. Eigenvectors should be found by a sequence of swap, combo, mult operations on the augmented matrix, followed by taking the partial ∂_{t_1} on invented symbol t_1 in the general solution to compute the eigenvector. In short, the eigenvectors are Strang's Special Solutions. In general there can be many eigenvectors for a single eigenvalue, however, for distinct eigenvalues there is exactly one eigenvector per eigenvalue.

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Ch7. (Linear Systems of Differential Equations)

The methods studied are (1) Linear Cascade method, (2) Cayley-Hamilton-Ziebur method and the 2×2 shortcut, (3) Eigenanalysis method, (4) Laplace's method, including the Laplace resolvent shortcut.

[30%] **Ch7(a)**: Solve for the general solution $x(t)$, $y(t)$ in the system below. Use any method that applies, from the lectures or any chapter of the textbook.

$$\begin{aligned}\frac{dx}{dt} &= x + y, \\ \frac{dy}{dt} &= 6x + 2y.\end{aligned}$$

Answer: Define $A = \begin{pmatrix} 1 & 1 \\ 6 & 2 \end{pmatrix}$. The eigenvalues $-1, 4$ are roots of the characteristic equation $\det(A - rI) = (r + 1)(r - 4) = 0$. By Cayley-Hamilton-Zeibur, $x(t) = c_1 e^{-t} + c_2 e^{4t}$. Using the first differential equation $x' = x + y$ implies $y(t) = x' - x = -2c_1 e^{-t} + 3c_2 e^{4t}$.

[30%] **Ch7(b)**: Consider the scalar system

$$\begin{cases} x' = 3x \\ y' = x, \\ z' = x + y \end{cases}$$

Report two possible methods that apply to solve for x, y, z . Then choose one method and display the solution details and the answer (details count 75%).

Answer:

Linear Cascade Method.

in progress The matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ 1 & 0 & 0 \\ 1 & 1 & 0 \end{bmatrix}$ is not diagonalizable, so only Laplace, linear cascade and Cayley-Hamilton-Ziebur apply.

[40%] **Ch7(c)**: Define

$$A = \begin{pmatrix} -3 & 4 & -10 \\ 0 & 2 & 0 \\ 5 & -4 & 12 \end{pmatrix}$$

The eigenvalues of A are 7, 2, 2. Apply the eigenanalysis method, which requires eigenvalues and eigenvectors, to solve the differential system $\mathbf{u}' = A\mathbf{u}$.

Answer: Form $A_1 = A - (7)I = \begin{pmatrix} -10 & 4 & -10 \\ 0 & -5 & 0 \\ 5 & -4 & 5 \end{pmatrix}$. Then $\text{rref}(A_1) = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. The last frame algorithm implies general solution of $A_1 \mathbf{x} = \mathbf{0}$ is $x_1 = -t_1$, $x_2 = 0$, $x_3 = t_1$. Take the partial on symbol t_1 to obtain the eigenvector $\mathbf{v}_1 = \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}$. Repeat with $A_2 = A - (2)I = \begin{pmatrix} -5 & 4 & -10 \\ 0 & 0 & 0 \\ 5 & -4 & 10 \end{pmatrix}$. Then $\text{rref}(A_2) = \begin{pmatrix} 1 & -\frac{4}{5} & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$. The last frame algorithm implies general solution of $A_2 \mathbf{x} = \mathbf{0}$ is $x_1 = 4t_1/5 - 2t_2$, $x_2 = t_1$, $x_3 = t_2$. Take the partial on symbols t_1, t_2 to obtain the eigenvectors $\mathbf{v}_2 = \begin{pmatrix} \frac{4}{5} \\ 1 \\ 0 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} -2 \\ 0 \\ 1 \end{pmatrix}$. The general solution of $\mathbf{u}' = A\mathbf{u}$ is $\mathbf{u}(t) = c_1 e^{7t} \mathbf{v}_1 + c_2 e^{2t} \mathbf{v}_2 + c_3 e^{2t} \mathbf{v}_3$.

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Ch9. (Nonlinear Systems)

[10%] **Ch9(a):** Which of the four types *center*, *spiral*, *node*, *saddle* can be asymptotically stable at $t = \infty$? Explain your answer.

Answer:

All except the center and the saddle. The center is stable but not asymptotically stable. All the others correspond to a general solution which can have an exponential factor e^{kt} in each term. If $k < 0$, then the solution limits to the origin at $t = \infty$.

In parts (b), (c), (d), (e) below, consider the **nonlinear system**

$$x' = 14x - 2x^2 - xy, \quad y' = 16y - 2y^2 - xy. \quad (1)$$

[20%] **Ch9(b):** Display details for finding the equilibrium points for the nonlinear system (1). There are four answers, one of which is (4, 6).

[20%] **Ch9(c):** Consider again system (1). Compute the Jacobian matrix at (x, y) . Then compute the Jacobian matrix at equilibrium point (4, 6).

[20%] **Ch9(d):** Classify the linear system for equilibrium (4, 6) as a node, spiral, center, saddle.

[30%] **Ch9(e):** Consider again system (1). What classification can be deduced for equilibrium (4, 6) of nonlinear system (1), according to the Pasting Theorem? Explain fully (details count 75%).

Answer:

in progress

The equilibria are constant solutions, which are found from the equations

$$\begin{aligned} 0 &= (14 - 2x - y)x \\ 0 &= (16 - 2y - x)y \end{aligned}$$

Considering when a zero factor can occur leads to the four equilibria (0, 0), (0, 8), (7, 0), (4, 6). The last equilibrium comes from solving the system of equations

$$\begin{aligned} 2x + y &= 14 \\ x + 2y &= 16 \end{aligned}$$

Linearization

The Jacobian matrix J is the augmented matrix of partial derivatives $\partial_x \vec{F}$, $\partial_y \vec{F}$ (column vectors) computed from

$$\vec{f}(x, y) = \begin{pmatrix} 14x - 2x^2 - xy \\ 16y - 2y^2 - xy \end{pmatrix}.$$

Then

$$J(x, y) = \begin{pmatrix} 14 - 4x - y & -x \\ -y & 16 - 4y - x \end{pmatrix}.$$

The four matrices below are $J(x, y)$ when (x, y) is replaced by an equilibrium point. Included in the table are the roots of the characteristic equation for each matrix and its classification based on the roots. No book was consulted for the classifications. The idea in each is to examine the limits at $t = \pm\infty$, then eliminate classifications. No matrix has complex eigenvalues, and that eliminates the center and spiral. The first three are stable at either $t = \infty$ or $t = -\infty$, which eliminates the saddle and leaves the node as the only possible classification.

$A_1 = J(0, 0) = \begin{pmatrix} 14 & 0 \\ 0 & 16 \end{pmatrix}$	$r = 14, 16$	node
$A_2 = J(0, 8) = \begin{pmatrix} 6 & 0 \\ -8 & -16 \end{pmatrix}$	$r = 6, -16$	saddle
$A_3 = J(28, 0) = \begin{pmatrix} -14 & -7 \\ 0 & 9 \end{pmatrix}$	$r = -14, 9$	saddle
$A_4 = J(12, 8) = \begin{pmatrix} -8 & -4 \\ -6 & -12 \end{pmatrix}$	$r = -10 - 2\sqrt{7}, -10 + 2\sqrt{7}$	node

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Ch10. (Laplace Transform Methods)

It is assumed that you know the minimum forward Laplace integral table and the 8 basic rules for Laplace integrals. No other tables or theory are required to solve the problems below. If you don't know a table entry, then leave the expression unevaluated for partial credit.

[40%] **Ch10(a)**: Fill in the blank spaces in the Laplace tables. Each wrong answer subtracts 3 points from the total of 40.

$f(t)$					
$\mathcal{L}(f(t))$	$\frac{2s}{1+s^2}$	$\frac{2}{3s+1}$	$\frac{5}{s^2+4}$	$\frac{2}{s^2} + \frac{1}{s}$	$\frac{1}{(s+1)^2+4}$

$f(t)$	t	$(1+e^{-t})^2$	$\cos(t)$	$e^{-t}\cos(t)$	$t\sin t$
$\mathcal{L}(f(t))$					

Answer: in progress

First table left to right: $\frac{113t^2}{2}$,

$e^{-4t/3}$,

$-2\sin 4t$,

$\pi \cos 2t$,

$e^{-(t-2)}u(t-2)$.

The unit step $u(t)$ is defined in the Edwards-Penney textbook, $u(t) = 1$ for $t \geq 0$, zero elsewhere.

Second table left to right: $\frac{1}{s^2}$,

$\frac{1}{(s-1)^2} + \frac{1}{s+1}$,

$\frac{s}{s^2+4} \Big|_{s \rightarrow (s-1)} = \frac{s-1}{(s-1)^2+4}$,

$\frac{6}{s^4} \Big|_{s \rightarrow (s+1)} = \frac{6}{(s+1)^4}$,

$-\frac{d}{ds} \frac{s}{s^2+1} = \frac{s^2-1}{(s^2+1)^2}$

[30%] **Ch10(b)**: Let u be the unit step, $u(t) = 1$ for $t \geq 0$, $u(t) = 0$ for $t < 0$. Compute $\mathcal{L}(x(t))$, given the mechanical problem

$$x''(t) + 4x(t) = 5(u(t-1) - u(t-2)), \quad x(0) = x'(0) = 0.$$

To save time, **do not solve** for $x(t)$.

Answer: in progress

Use $f(t) = 100(u(t-2) - u(t-3))$ and the second shifting theorem. Then $\mathcal{L}(100u(t-2)) = e^{-2s}\mathcal{L}(100|_{t \rightarrow t+2}) = e^{-2s}\mathcal{L}(100) = 100e^{-2s}/s$. Similarly, $\mathcal{L}(100u(t-3)) = 100e^{-3s}/s$. The answer: $\mathcal{L}(f(t)) = 100(e^{-2s} - e^{-3s})/s$.

[30%] **Ch10(c)**: Solve for $g(t)$ in the equation $\mathcal{L}(g(t)) = \frac{e^{-2s}}{s+5}$.

Answer: in progress

First write $\frac{1}{s^2+9} = \mathcal{L}(\frac{1}{3} \sin(3t))$ using the backward Laplace table. Apply the second shifting theorem: $e^{-as} \mathcal{L}(h(t)) = \mathcal{L}(h(t-a) \mathbf{u}(t-a))$. Then $\mathcal{L}(f(t)) = e^{-\pi s} \mathcal{L}(\frac{1}{3} \sin(3t)) = \mathcal{L}(\frac{1}{3} \sin(3z) u(z) \Big|_{z=t-\pi}) = \frac{1}{3} \mathcal{L}(\sin(3t-3\pi) u(t-\pi))$. Lerch's theorem implies $f(t) = \frac{1}{3} \sin(3t-3\pi) u(t-\pi) = -\frac{1}{3} \sin(3t) u(t-\pi)$.