south pole of Mars," *Icarus*, in press.
[7] R. H. Selzer, "Digital computer processing of X-ray photographs," in *Proc. Rochester Conf. on Data Acquisition and Processing in Biology and Medicine* (Rochester, N. Y., July 27, 1966), vol. 5. New York: Pergamon, 1968, pp. 309–325.
[8] R. H. Selzer, "Use of computers to improve biomedical image quality," in *1968 Fall Joint Computer Conf., AFIPS Conf. Proc.*, vol. 33, Pt. I. Washington, D. C.: Thompson, 1968, pp. 817–834.
[9] R. H. Selzer, "Recent progress in computer processing of X-ray and radioisotope scanner images," *Biomed. Sci. Instrum.*, vol. 6, pp. 225–234, 1969.
[10] S. D. Rockoff and R. H. Selzer, "Radiographic trabecular quantitation of human lumbar vertebrae in situ," in *Proc. Conf. on Progress in Methods of Bone Mineral Measurements* (Bethesda, Md., 1968, NIH, NIAMS), pp. 331–351.
[11] S. H. Brooks, R. H. Selzer, D. W. Crawford, and D. H. Blankenhorn, "Computer image processing of peripheral vascular angiograms," in preparation.
[12] B. C. Bridges, *Practical Fingerprinting*. New York: Funk and Wagnalls, 1942, 374 pp.
[13] H. Cummins and M. Charles, *Fingerprints, Palms and Soles*. New York: Dover, 1961, pp. 319.
[14] R. J. Blackwell, "Fingerprint image enhancement by computer methods," presented at 1970 Carnahan Conf. Electron. Crime Countermeasures (Lexington, Ky., Apr. 17, 1970).
[15] I. S. Bowen, "The 200 inch Hale telescope," in *Telescopes*, vol. I, *Stars and Stellar Systems*, G. P. Kuiper and B. M. Middlehurst, Eds. Chicago, Ill.: Univ. Chicago, 1960, pp. 1–15.
[16] E. L. O'Niell, *Introduction to Statistical Optics*. New York: Addison-Wesley, 1963, pp. 86, 99.

Image Processing in the Context of a Visual Model

THOMAS G. STOCKHAM, JR., MEMBER, IEEE

***Abstract*—A specific relationship between some of the current knowledge and thought concerning human vision and the problem of controlling subjective distortion in processed images are reviewed.**

I. Introduction

IMAGE QUALITY is becoming an increasing concern throughout the field of image processing. The growing awareness is due in part to the availability of sophisticated digital methods which tend to highlight the need for precision. Also there is a developing realization that the lack of standards for reading images into and writing images out of digital form can bias the apparent effectiveness of a process and can make uncertain the comparison of results obtained at different installations. Greater awareness and the desire to respond to it are partially frustrated, because subjective distortion measures which work well are difficult to find. Part of the difficulty stems from the fact that physical and subjective distortions are necessarily different.

The ideas presented here spring from our reevaluation of the relationship between the structure of images and 1) the problem of quantitative representation, 2) the effect of desired processing and/or unwanted distortion, and 3) the interaction of images with the human observer. They provide a framework in which we think about and perform our image processing tasks. By adding to our understanding of what is to be measured when dealing with images and by strengthening the bridge between the objective (physical) and the subjective (visual) aspects of many image processing issues, these ideas have clarified the meaning of image quality and thus have enhanced our ability to obtain it. We offer them with the hope that they may aid others as well.

In the course of the discussion it is noted that image processors which obey superposition multiplicatively instead of additively, bear an interesting resemblance both operationally and structurally to early portions of the human visual system. Based on this resemblance a visual model is hypothesized, and the results of an experiment which lends some support to and provides a calibration for the model are described. This tentative visual model is offered only for its special ability to predict approximate visual processing characteristics. (See footnote 11.)

In recent years there has been a large amount of quantitative work done by engineers and scientists from many fields

Manuscript received January 31, 1972; revised April 20, 1972. This research was supported in part by the University of Utah Computer Science Division monitored by Rome Air Development Center, Griffiss Air Force Base, N. Y. 13440, under Contract F30602-70-C-0300, ARPA order number 829.

The author is with the Computer Science Division, College of Engineering, University of Utah, Salt Lake City, Utah 84112.

in support of a model for human vision. While many of these works are not referenced explicitly here, we have attempted to reference papers and texts which do a good job of collecting these references in a small number of places while providing a unifying interpretation [1]–[5].

II. Some Philosophy about Image Processing

The notion of processing an image involves the transformation of that image from one form into another. Generally speaking, two distinct kinds of processing are possible. One kind involves a form of transformation for which the results appear as a new image which is different from the original in some desirable way. The other involves a result which is not an image but may take the form of a decision, an abstraction, or a parameterization. The following discussion limits itself primarily to the first kind of processing.

The selection of a processing method for any particular situation is made easier when the available processes have some kind of mathematical structure upon which a characterization of performance can be based. For example, the bulwark for most of the design technology in the field of signal processing is the theory of linear systems. The fact that the ability to characterize and utilize these systems is as advanced as it is, stems directly from the fact that the defining properties of these systems guarantee that they can be analyzed. These analyses, based on the principle of superposition, lead directly to the concepts of scanning, sampling, filtering, waveshaping, modulation, stochastic measurement, etc.

Equally important, however, is the idea that the mathematical structure of the information being processed be compatible with the structure of the processes to which it is exposed. For example, it would be impossible to separate one radio transmission from another if it were not for the fact that the linear filters used are compatible with the additive structure of the composite received signal.

In the case of images the selection of processing methods has often been based upon tradition rather than upon a consideration of the ideas given above. In fields such as television and digital image processing where electrical technology is a dominating influence, the tradition has centered around the use of linear systems.

This situation is a very natural one since the heritage of electrical image processing stems from those branches of classical physics which employ linear mathematics as their foundation. Specifically, it is interesting to follow the development from electromagnetic field theory to electric measurements, circuit theory, electronics, signal theory, communications theory, and eventually to digital signal processing. The situation is similar when considering the role of optics in image processing, the laws of image formation and degradation being primarily those determined from linear diffraction theory.

The question that arises is whether this tradition of applying linear processing to images is in harmony with the ideas given above. The major point at issue cannot be whether the processors possess enough structure, because linear systems certainly do. The issue is then whether that structure is compatible with the structure of the images themselves. To clarify this issue the question of image structure must be elaborated upon.

III. The Structure of Images

As an energy, signal light must be positive and nonzero. This situation is expressed in (1)

$$\infty > I_{x,y} > 0 \tag{1}$$

where I represents energy, or intensity as it is commonly called, and x and y represent the spatial domain of the image. Furthermore, since images are commonly formed of light reflected from objects, the structure of images divides physically into two basic parts. One part is the amount of light available for illuminating the objects; the other is the ability of those objects to reflect light.

These basic parts are themselves spatial patterns, and like the image itself must be positive and nonzero as indicated in (2) and (3)[1]

$$\infty > i_{x,y} > 0 \tag{2}$$

$$1 > r_{x,y} > r_{\min} \approx 0.005. \tag{3}$$

These image parts, called the illumination component and the reflectance component, respectively, combine according to the law of reflection to form the image $I_{x,y}$. Since that law is a product law, (2) and (3) combine as in (4)

$$\infty > I_{x,y} = i_{x,y} \cdot r_{x,y} > 0 \tag{4}$$

which is in agreement with (1).

It follows from (4) that two basic kinds of information are conveyed by an image. The first is carried by $i_{x,y}$, and has to do primarily with the lighting of the scene. The second is carried by $r_{x,y}$, and concerns itself entirely with the nature of the objects in the scene. Although they are delivered in combination, these components are quite separate in terms of the nature of the message conveyed by each.

So far it has been assumed that the process of forming an image is carried out perfectly. Since ideal image forming methods do not exist and can only be approached, a practical image will only approximate that given in (4). Because most image forming methods involve linear mechanisms such as those which characterize optics, a practical image can be regarded as an additive superposition of ideal images. This fact is expressed in (5)

$$\infty > \tilde{I}_{x,y} = \int_{-\infty}^{\infty} I_{X,Y} h_{x,X;y,Y} \, dX dY > 0 \tag{5}$$

where $\tilde{I}_{x,y}$ represents a practical image and $h_{x,X;y,Y}$ represents the so-called point spread function of the linear image forming mechanism. In other words $h_{x,X;y,Y}$ is the practical image that an ideal image consisting of a unit intensity point of light located at $x=X$ and $y=Y$ would produce. Obviously h must be nonnegative.

If the point spread function is the same shape for all points of light in the ideal image, then the superposition integral (5) becomes a convolution integral (6)

$$\infty > \tilde{I}_{x,y} = \int_{-\infty}^{\infty} I_{X,Y} h_{x-X;y-Y} \, dX dY > 0 \tag{6}$$

[1] It is almost impossible to find a material that reflects less than about 1 percent of the incident light.

which is conventionally expressed using a compact notation as in (7)

$$\infty > \bar{I}_{x,y} = I_{x,y} * h_{x,y} > 0. \tag{7}$$

Combining (4) and (7) we obtain (8)

$$\infty > \bar{I}_{x,y} = (i_{x,y} \cdot r_{x,y}) * h_{x,y} > 0 \tag{8}$$

which under the assumption of a position invariant point spread function summarizes the essential structure of practical images as they are considered in most current efforts.

The expression (8) places in evidence the three essential components of a practical image. If $h_{x,y}$ is sufficiently small in its spatial extent, the practical image can be taken as an adequate approximation to the ideal. If $h_{x,y}$ fails in this respect, the practical image can be processed by any one of a variety of methods in an attempt to remedy the situation.[2]

Since the objective of the present discussion focuses primarily on the structure of an ideal image, it will be assumed in the following that the effect of $h_{x,y}$ can be neglected.[3] Primary concern here is thus redirected to (4).

We now return to the issue posed at the end of Section II as to whether or not the mathematical structure of linear processors is compatible with the structure of the images themselves. Since (4) indicates that the image components are multiplied to form the composite, and further since linear systems are compatible with signals possessing additive structure, it follows that there exists basic incompatibility. However, this incompatibility depends in a basic way upon some implicit assumptions which have been imposed upon the structure as described in (4).

An essential ingredient to the structure of images as expressed in (4) is the assumption that an image is an energy signal. This assumption really amounts to a choice of a representation for an image. The nature of that choice can be extremely important. To clarify this concept the question of representation must be elaborated upon.

IV. The Representation of Images

A key question in the transmission, storage, or processing of any information is that of representation. The reason that the choice of representation is important is that the problems of transmission, storage, and processing can be substantially effected by it.

If an ideal physical image is considered as a carrier of information, it follows that nature has already chosen a representation. It takes the form of light energy. Furthermore, if one takes nature literally when sensing an optical image, one will continue that representation by creating a signal proportional to the intensity of that light energy. Indeed this representation seems like a very natural one, and in fact as already indicated, it is commonly used in television and digital image processing.

Strangely enough representation by light intensity analogy is a relatively new practice in image technology. The process of photography, now over a century old, does not use it. It has only been with the advent of electrical imaging methods that it has received attention.

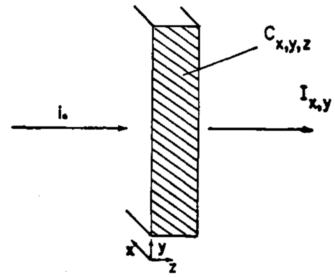


Fig. 1. An intensity image $I_{x,y}$ as reproduced by the transmission of light through a volume concentration of amorphous silver $C_{x,y,z}$.

In order to clarify this point, imagine a black and white photographic transparency which portrays some optical image. In order to see the reproduction one must illuminate the transparency uniformly with some intensity i_0 and somehow view the transmitted pattern of light intensity $I_{x,y}$. The quantities of light which are transmitted are determined by the volume concentrations of amorphous silver suspended in a gelatinous emulsion. Thus it is these concentrations which represent the image in its stored form. Let these concentrations be expressed as $C_{x,y,z}$.

Physically the situation is as depicted in Fig. 1. In order to derive the relationship between the reproduced image $I_{x,y}$ and $C_{x,y,z}$ we must consider the transmission of light through materials. The physics of the situation is given in (9)

$$\frac{di}{dz} = -kC_{x,y,z}i \tag{9}$$

where i is the intensity of the light at any point in the transmitting material and k is a constant representing the attenuating ability of a unit concentration of amorphous silver. Integration of (9) according to standard methods yields (10)

$$\int_{i_0}^{I_{x,y}} \frac{di}{i} = -k\int_0^{z_t} C_{x,y,z}dz \tag{10}$$

where z_t represents the thickness of the emulsion. Since the integral in the right-hand side of (10) represents the total quantity of silver per unit area of the transparency independent of how that silver is distributed in the z dimension, (10) can be rewritten as in (11)

$$\ln (I_{x,y}/i_0) = -kd_{x,y}. \tag{11}$$

A solution of (11) for $I_{x,y}$ yields (12)

$$I_{x,y} = i_0e^{-kd_{x,y}}. \tag{12}$$

From (11) it can be seen that in the case of a photographic transparency, the physical representation of the image is actually $d_{x,y}$ which is proportional to the logarithm of the reproduced intensity image. In turn (12) reveals that the physical representation $d_{x,y}$ is exponentiated during its conversion to light intensity. Further, it follows that if $I_{x,y}$ is a faithful reproduction of the original intensity image from which the transparency was made, then the quantities of silver used to form the representation $d_{x,y}$ must have been

[2] For an excellent and recent summary, bibliography, and set of references representative of the many interesting efforts in this area, see Section II of a recent article by Huang *et al.* [1].

[3] There is still much to be learned both practically and theoretically about restoring practical images to the point where this is possible. Such restoration methods are very important; and since they attempt in part to compensate for distortions caused by linear mechanisms, linear processing is used extensively and often with great success.

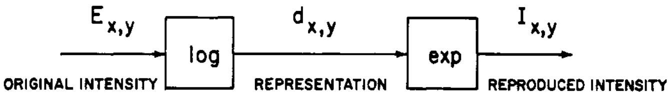


Fig. 2. In photography an image is represented by the total quantity $d_{x,y}$ of amorphous silver per unit image area. For faithful reproduction $d_{x,y}$ must be proportional to the logarithm of the image intensities.

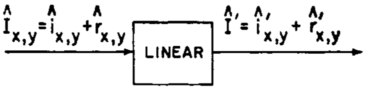


Fig. 3. A density image as processed by a linear system. Note that the basic structure of the image is preserved. The output is a processed illumination plus a processed reflectance regardless of what the process may be.

deposited in the emulsion by a process which was logarithmically sensitive to light energy.

This situation is summarized in Fig. 2 where the logarithmic and exponential transformations which mechanize the formation of a photographic image are placed in evidence. The variables i_0 and k which appear in (11) and (12) have been omitted for convenience since they are only scaling constants.[4]

The relationship of (12) is well known in photography but is usually presented in a somewhat altered form as in (13).

$$\log_{10} (i_0/I_{x,y}) = D_{x,y}. \tag{13}$$

Here the quantity $D_{x,y}$, called density, is proportional to $d_{x,y}$ but related directly to the common logarithm in a manner similar to that used in the definition of the decibel. Because $d_{x,y}$ and $D_{x,y}$ are both related to the popular notion of density it is reasonable to call any logarithmic representation of an image a density representation. As indicated above, all such representations are the same except for the choice of the two constant parameters.

Taking this into account (11) and (12) may be generalized to (14) and (15)

$$\hat{I}_{x,y} = \log (I_{x,y}) \tag{14}$$

$$I_{x,y} = \exp (\hat{I}_{x,y}) \tag{15}$$

where the hatted variables represent density and the unhatted variables represent intensity. All density representations are the same except for a scale factor and an additive constant.

V. Relationships Between Processing, Structure, and Representation

A study of the use of a density representation for images leads to a chain of interesting observations. These observations begin with the introduction of density representations into the previous discussion concerning the structure of ideal images. This introduction changes (1)–(4)[5]

$$\infty > \hat{I}_{x,y} = \log (I_{x,y}) > -\infty \tag{16}$$

$$\infty > \hat{\imath}_{x,y} = \log (i_{x,y}) > -\infty \tag{17}$$

$$0 > \hat{r}_{x,y} = \log (r_{x,y}) > \hat{r}_{\min} \tag{18}$$

and

$$\infty > \hat{I}_{x,y} = \hat{\imath}_{x,y} + \hat{r}_{x,y} > -\infty \tag{19}$$

where $\hat{\imath}_{x,y}$ and $\hat{r}_{x,y}$ represent illumination[6] and reflection densities, respectively.

It is obvious from these equations that a change from an energy representation to a density representation has introduced some interesting changes in the apparent structure of images. There is no longer a restriction upon the range of the representation. To see this fact compare (1) with (16). The manner in which the basic components of the scene are combined has been changed from multiplication to addition (compare (4) and (19)). Finally, the scene components themselves have been changed from an energy representation to a density representation.

In the case of the reflection component the transformation to a density representation is a very satisfactory one. This is so, because to a great extent the physical properties of an object which determine its ability to reflect light are the densities of the light blocking materials from which it is formed. The situation is similar to that of the photographic transparency as described in (9)–(12). Thus by using (19) the physical properties of an object are represented more directly than in (4).

The single most important effect of using a density representation is that it makes the structure of images compatible with the mathematical structure of linear processing systems. This fact is true, because linear systems obey additive superposition and from (19) we see that the basis for the structure of a density representation of an image is additive superposition.

To build upon this observation consider Fig. 3 in which a density image is being processed by a linear system. The input of the system is given as in (19). It follows from the property of superposition in linear systems that the output must be given in (20)

$$\infty > \hat{I}_{x,y}' = \hat{\imath}_{x,y}' + \hat{r}_{x,y}' > -\infty \tag{20}$$

where the primes indicate processed quantities. But (21) is in the same form as (19). What (20) says is that the basic structure of a density image is preserved by any linear processor. More specifically the illumination component of the processed image *is* the processed illumination component and the reflection component of the processed image *is* the processed reflection component.

For comparison consider the effect of a linear system upon an intensity image. The input is given in (4). It is clear that the notion of structure preservation cannot be maintained in this case. What is even more embarrassing is the fact that there is little guarantee that the output will be positive and nonzero which it must if it is to be regarded as an image at all.

Because an image carries information, and because information can be measured using concepts of probability, it is interesting to consider the probability density functions

[4] Actually i_0 is just a constant of proportionality on the image intensity and can be neglected if one considers normalized images only. Also k can be absorbed into the logarithmic and exponential transformations by adjusting the base being used.

[5] The minimum reflection density using the common logarithm would almost never exceed 2.0. See footnote 1.

[6] The concept of an illumination density may seem strange at the outset but proves to be an important mathematical concept even though it may be difficult to assign it any physical significance.

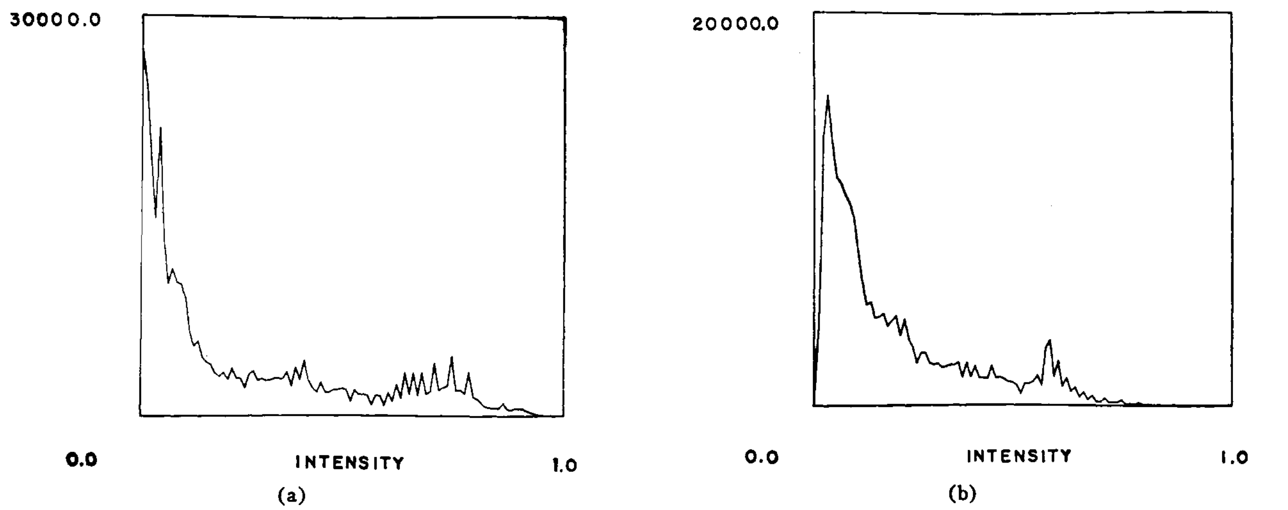


Fig. 4. Intensity histograms of 100 bins each obtained from high quality images carefully digitized to 340 by 340 samples using 12 bit/sample. (a) Three wide dynamic range scenes. (b) Two scenes of less dynamic range (approx. 30:1).

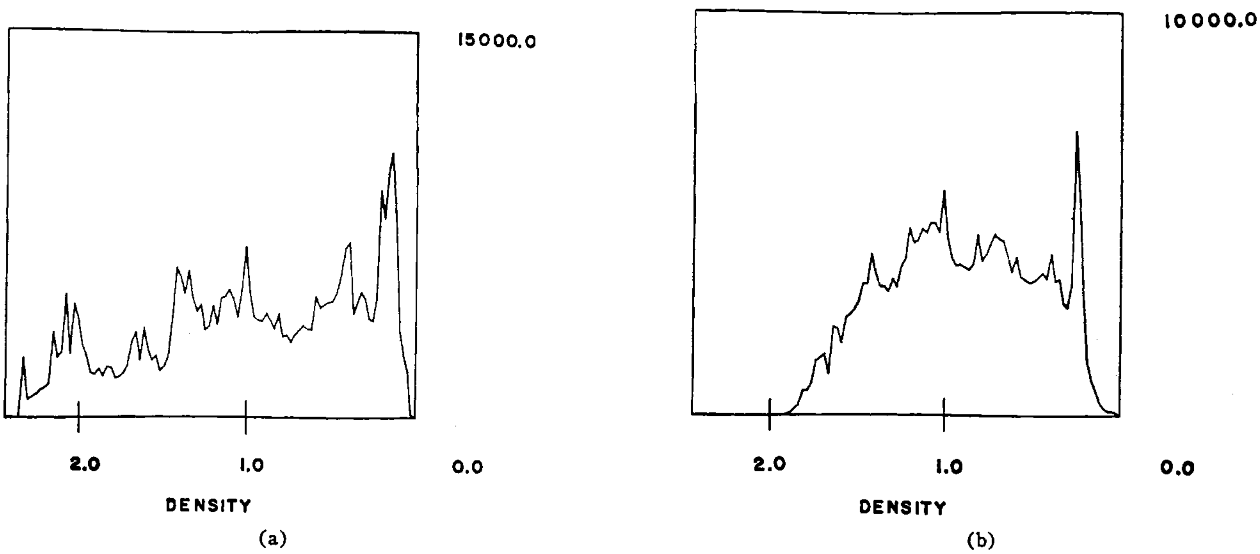


Fig. 5. Density histograms of 100 bins each obtained from the same images as in Fig. 4.

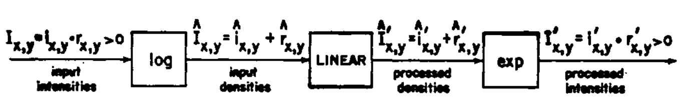


Fig. 6. An intensity image as processed by a multiplicative system. Again the basic structure of the image is preserved and the output is a processed illumination times a processed reflectance.

which are associated with both forms of representation. To this end Fig. 4 shows histograms for images which were represented by intensities and Fig. 5 shows histograms for the same images as represented by densities. These images were obtained using very careful methods from very high quality digital images.

It is instructive to compare the highly skewed distributions of Fig. 4 with the more nearly symmetric ones of Fig. 5. The fact that a density representation of an image tends to fill the representation space more uniformly than an intensity representation implies some important advantages for the former. For example, consider the problem of digitizing either representation by means of a quantizer using a binary code. The nearly symmetric distributions of Fig. 5 imply a more efficient use of the information carrying capacity of the binary code, a rectangular distribution being ideal in this respect. In addition, the symmetric distributions are more nearly aligned with the conventional assumptions associated with signals in many theoretical studies.

VI. Multiplicative Superposition in Image Processors

For some purposes it is important to be able to think of an image as represented by intensities. It is absolutely essential to do so when sensing an image to begin with or when reproducing an image for observation. In these cases it is possible to retain the match between the structure of images and the structure of processors by combining the concepts embodied in Figs. 2 and 3. This situation is depicted in Fig. 6. The input is given as in (4). It follows from (20) and (15) that

$$\infty > I_{x,y}' = \exp(\hat{I}_{x,y}') = \exp(\hat{i}_{x,y}' + \hat{r}_{x,y}') > 0 \quad (21)$$

which by the properties of the exponential function becomes

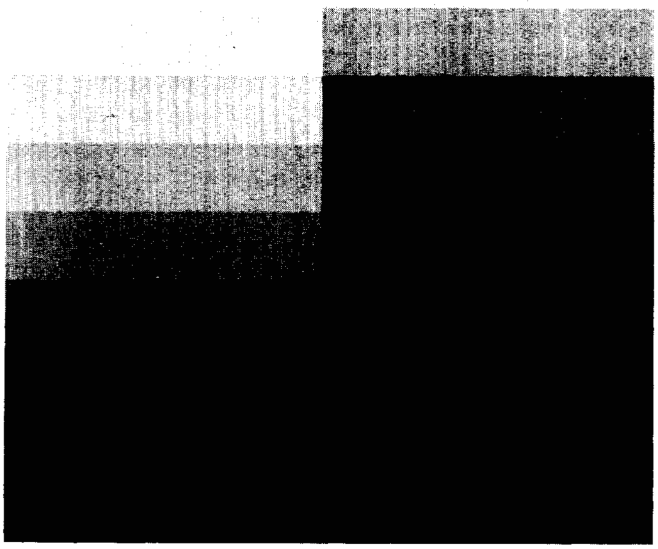


Fig. 7. Two grayscales.[8] (a) Linear intensity steps. (b) Linear density steps.

$$\infty > I_{x,y}' = \exp(\hat{\imath}_{x,y}') \cdot \exp(\hat{r}_{x,y}') > 0. \tag{22}$$

But in analogy with (21) we have

$$i_{x,y}' = \exp(\hat{\imath}_{x,y}') \tag{23a}$$

and

$$r_{x,y}' = \exp(\hat{r}_{x,y}'). \tag{23b}$$

So substituting (23) into (22) we get

$$\infty > I_{x,y}' = i_{x,y}' \cdot r_{x,y}' > 0 \tag{24}$$

which is in the same form as (4).

Again the basic structure of the image is preserved. However, this time the multiplicative superposition which characterizes the structure of an intensity image is compatible with the mathematical structure of the processor of Fig. 6. It follows that Fig. 6 depicts a class of systems which obey multiplicative superposition [2]. Besides demonstrating the preservation of structure for intensity images (24) also reveals the fact that a multiplicatively processed image is itself positive and nonzero and thus realizable. This later observation transcends the fact that the system used to process the input densities in Fig. 6 is linear, because the processed intensities are formed by exponentiating the processed densities regardless of how those densities were produced. The result of exponentiating a real density is always positive and nonzero. This property of density processing is called the realizable output guarantee.

VII. Multiplicative Superposition in Vision

Although a great deal of sophisticated and elaborate knowledge has been gained in the last several decades about the problem of communicating electrically between various sorts of automatic mechanisms, dissappointingly little has been done to match the ultimate source and receiver, namely the human being, to this body of knowledge and these systems. The basic obstacles have been a lack of understanding of the human mechanisms in terms describable by the available theory and the difficulty in studying the human mechanisms which are involved.

The philosophy that any communications system, whether man-made or natural, has structure and that that structure should be matched to the communications task at hand, seems to provide a stepping stone for understanding the operation of some of these systems. In this regard we would like to take the concept of a multiplicative image processor and explore its possible relationship to the known properties of early portions of the human visual system.

In many respects the multiplicative image processors previously described and their canonic form as represented in Fig. 6 bear an interesting resemblance to many operational characteristics of the human retina.[7] The presence of an approximately logarithmic sensitivity in vision has been known for some time [3]. Even more readily evident, and mechanized through the process of neural interaction, is the means for linear filtering [3], [4].

A. Logarithmic Sensitivity

The fact that light sensitive neurons fire at rates which are proportional to the logarithm of the light energy incident upon them has been measured for simple animal eyes [3, pp. 246–253]. Similar experiments with human beings are inconvenient to say the least, but there are some interesting experiments that serve as a partial substitute. The most convincing of these is the so called "just noticeable difference" experiment [5]. In this experiment an observer is asked to adjust a controllable light patch until it is just noticably brighter or darker than a reference light patch. The experimenter then steps his way through the gamut of light intensities from very bright to very dark. The step numbers are then plotted as a function of the intensity of the reference light. The resulting curve is very close to logarithmic over several orders of magnitude of intensity.

For a direct but less objective demonstration of this relationship consider the gray-scale steps[8] presented in Fig. 7. In Fig. 7(a) the scale consists of equally spaced intensity steps. In Fig. 7(b) the scale consists of exponentially spaced intensity steps which is the same as equally spaced density steps. The scale in Fig. 7(b) appears as a more nearly equally spaced scale than that of Fig. 7(a) so that the eye appears to respond more nearly to densities than to intensities.

B. Linear Filtering through Neural Interaction

The mechanism for linear spatial processing in vision is observed in the Hartline equations [4, pt. I, ch. 3], [3, ch. 11, pp. 284–310]. The effect of this processing can be observed by means of a number of simple optical illusions.

The simplest of these illusions is known as the illusion of simultaneous contrast[9] and can easily be observed in Fig. 8. In this image we observe two small squares surrounded by larger rectangles, one light, one dark. In fact the two small

[7] A recent, lucid, and elaborate discussion of these characteristics is presented by Cornsweet [3]. See especially chs. XI and XII.

[8] This and several other test images shown here should be presented using a calibrated display or calibrated photography. An uncertain but considerable distortion will have taken place during the printing of this paper. The reader must take this into account and estimate the possible degradation for himself.

[9] For a more complete discussion see [3, pp. 270–284].

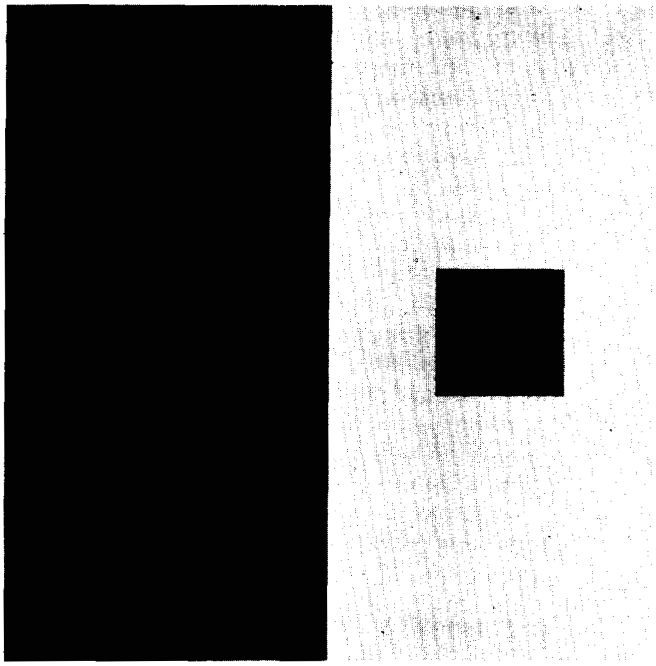


Fig. 8. The illusion of simultaneous contrast. The two small squares are of exactly the same intensity.

squares are exactly the same shade of gray. They appear different, however, due to their surroundings. This illusion can be explained at least qualitatively by assuming that the image has been subject to linear spatial filtering in which low spatial frequencies have been attenuated relative to high spatial frequencies. Filters of this type cause the averages of different areas in one image to seek a common level. Since in Fig. 8 the area of the left has a darker average, it will be raised, making the left square brighter. Likewise, since the area on the right has a lighter average, it will be lowered, making the right square less bright.

Another illusion can be observed by returning attention to Fig. 7(b). Each rectangle in this gray scale is one uniform shade of gray. However, each rectangle appears to be darker near its lighter partner and lighter near its darker partner. Again the phenomenon can be explained at least qualitatively by the assumption of linear spatial filtering.[9]

The final illusion to be discussed here is presented in Fig. 9. It is known as the illusion of Mach bands [3, pp. 270–284], [4]. In this image[8] there are two large areas, one light and one dark but each of a uniform shade. These two areas are coupled by a linearly increasing density wedge (exponentially increasing intensity wedge) as indicated in Fig. 9(b). The observer will notice that immediately at the left and at the right of this wedge are a dark and light band as implied by Fig. 9(c). These bands, known as Mach bands, can also be explained at least qualitatively by linear processing.[10]

C. Saturation Effects

So far this discussion has implied that the linear spatial processing of densities can explain a number of visual phe-

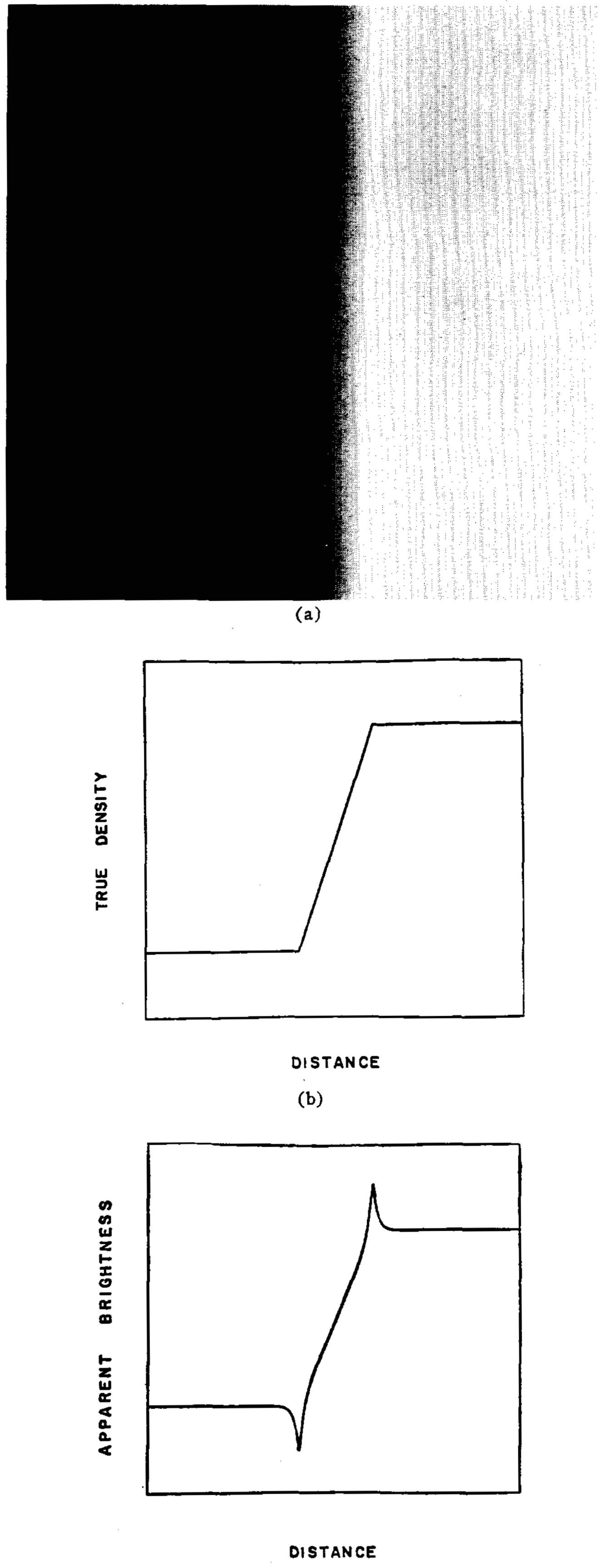


Fig. 9. The illusion of Mach bands. (a) Observe the dark and light bands which run vertically at the left and right of the ramp, respectively. (b) The true density representation of the image. (c) The approximate apparent brightness of the image.

[10] Quantitative studies of this illusion are common. Unfortunately, almost all of them employ a matching field or light which in turn perturbs the measurement considerably. Mach himself warned of this problem [4, pp. 50–54, 262, 305, 322] and suggested that there is no solution. The psychophysical experiment to be described later is offered as a possible counter example to this suggestion.

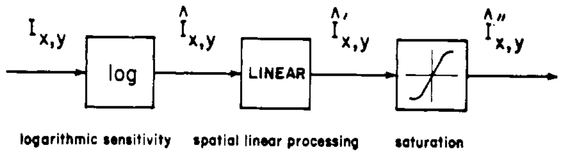


Fig. 10. A possible approximate model for the processing characteristics of early portions of the human visual system.

nomena. It is clear that these visual phenomena are only observable if there is a proper amount of light available for their presentation. It is common knowledge that below certain illumination levels one cannot see well if at all. The same is true if illumination levels become too great.

The physical limitations of any visual mechanism guarantee that saturation or threshold effects will occur if intensity levels are raised or lowered far enough. In this respect any consideration of the relationship between the processing of densities and properties of vision must eventually include the effects of saturation.

D. *A Process Model for Early Portions of the Human Visual System*

The preceding discussions suggest a model for the processing characteristics of early portions of the human visual system.[11] This model is shown in Fig. 10. The output $\hat{I}_{x,y}''$ is a saturated version of a linearly processed density representation. The linear processing is presumably of the form in which low spatial frequencies are attenuated relative to high spatial frequencies.

The most useful implications of this model do not come from its relationship to the optical illusions which we have already discussed as much as from the operational characteristics it embodies. The operational characteristics in question center around the ability of the human visual system to maintain its sensitivity to patterns of relatively low contrast in the context of a total image in which intensities are spread across a very large dynamic range,[12] and its ability to preserve an awareness of the true shades of an object in spite of huge differences in illumination. Moreover, these abilities are embodied without sacrificing the basic structure of images with respect to the separate physical components of illumination and reflectance!

If the illumination component of an image did not vary in space, (4) would become

$$I_{x,y} = i \cdot r_{x,y}. \tag{25}$$

In this case[13] the dynamic range of an image would be limited to about 100:1, because it would be determined by the reflection component[1] alone. Problems with saturation effects would be relieved if not avoided altogether. In addition the true shade of an object would be reproduced directly by $I_{x,y}$.

[11] This model is representative of approximate processing characteristics at early stages only. It is not intended as a biophysical or anatomical model for any specific visual mechanism or as an exact or complete processing representation. In image processing some such model must be assumed even if it is by default. The classical default assumption is that of fidelity reproduction namely that like an ideal camera the eye "sees" what it sees.

[12] The dynamic range of an image is the ratio of the greatest to the least intensity value therein contained. Ratios in excess of 1000:1 are often encountered by the eye or camera.

[13] This configuration, often sought at great expense in photographic and television studios, is called flat lighting.

Unfortunately, the illumination component of an image varies a great deal, often more than the reflectance component. For example a black piece of paper in bright sunlight will reflect more light than a white piece of paper in shadow. In the proper environment both situations could occur in the same image at the same time, but an observer would always call the white paper "white" and the black paper "black" in spite of the fact that the black paper would be represented by a higher intensity than the white paper. This visual phenomenon is called brightness constancy. Moreover, if there were low contrast markings on either sheet of paper they could be read in spite of their insignificance with respect to the total intensity scale.

With these facts in mind it is interesting to note that the system of Fig. 10 tends to produce an output in which the variations in illumination are indeed reduced. This is so, because the illumination component dominates the Fourier spectrum of a density image at low spatial frequencies while the reflectance component dominates at high spatial frequencies. As a result, the spatial linear filtering previously described reduces the illumination variations, because it attenuates low frequencies relative to high frequencies. At the same time the basic structure of images is preserved because the model operates linearly on a density representation.

The detailed consequences of this situation are described in more detail in [2, sec. V]. There the use of multiplicative processors for the purpose of simultaneous dynamic range reduction and detail contrast enhancement is discussed and demonstrated. An example of an image possessing some serious dynamic range problems is shown in Fig. 11 before and after such processing. Notice how the illumination is extremely variable from the outside to the inside of the building. In the unprocessed image, details within the room though present in the original are obscured by the limited dynamic range capabilities of the printing process you are now viewing. In the processed image these details are present in spite of this limitation.

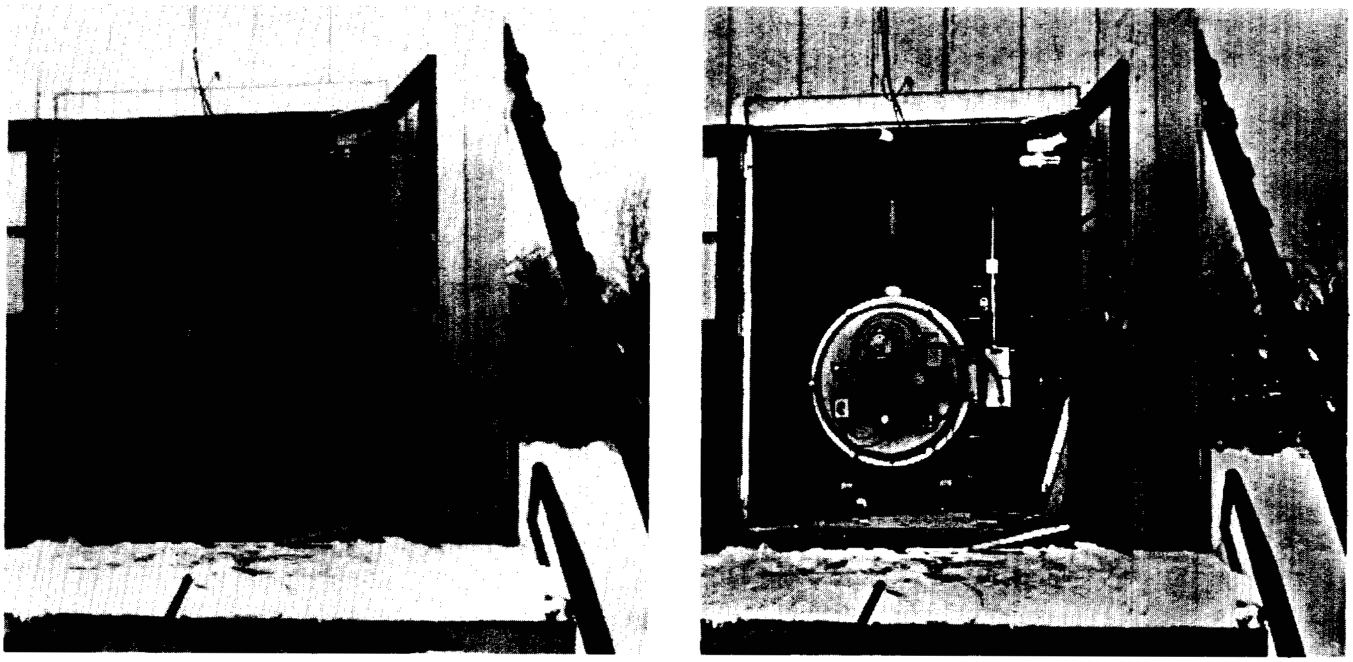
E. *Model and Process Compatibility*

When the image of Fig. 11(b) is observed, the total processing system including the approximate visual model is that shown in Fig. 12 which combines Figs. 6 and 10. In Fig. 12(a) the two linear systems which characterize the processor and the visual system are labeled H and V, respectively. Fig. 12(b) shows the simplified exact equivalent system in which as much merging of subprocesses as is possible has been performed. The new composite linear system labeled $H \cdot V$ is merely the cascade of the two previous ones.

Fig. 12(b) demonstrates the compatibility of the visual model and the multiplicative image processor. It does so by placing in evidence the fact that within the validity of the model the experience of viewing a processed image is indistinguishable from that of viewing an unprocessed image except that it is possible to alter the linear processing performed through the manipulation of the linear system labeled H.

F. *Model Testing and Calibration*

The approximate visual model of Fig. 10 has been motivated in the above by studying certain illusions, noting certain aspects of neural structure and neural measurement, and by concentrating attention upon certain desirable and available performance characteristics. This motivation can be sup-



(a) (b)

Fig. 11. A large dynamic range scene. (a) Before processing. (b) After processing with a multiplicative processor adjusted to attenuate low and to amplify high frequency components of density. (Note: These and all other images in this paper are digital.)

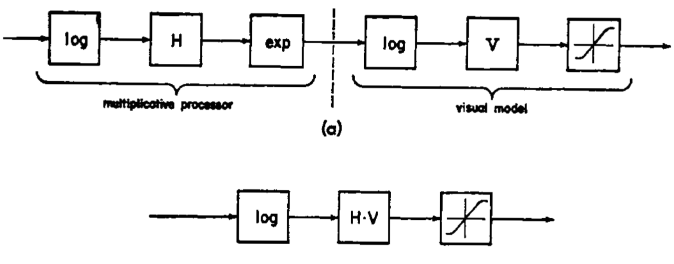


Fig. 12. Total processing system including visual model when viewing Fig. 11(b). (a) Unsimplified system. Processed intensities appear at the vertical dotted line. (b) Simplified system with processors merged.

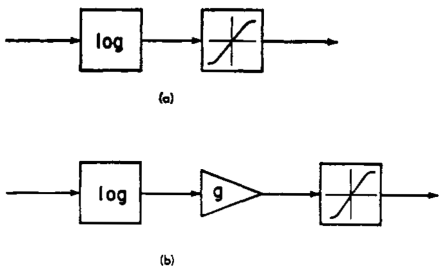


Fig. 13. Total processing system when viewing an image which has been subject to a multiplicative processor the linear component of which has been adjusted to be the inverse of the linear component of the visual model. (a) H is exactly the inverse of V. (b) H is the inverse of V except for a constant of proportionality g.

ported by a testing experiment which is suggested by the situation depicted in Fig. 12. If the system H were adjusted to become the inverse of the system V, the system of Fig. 12(b) could be further simplified as shown in Fig. 13. In this situation it should not be possible to observe the optical illusions described above and portrayed in Figs. 8 and 9.

An experiment designed to find an H which would simultaneously cancel the optical illusions described above can be carried out with significant success [6]. By comparing the pattern of Fig. 14 with Figs. 8 and 9 one can see that this pattern strongly induces the illusions in question.[8] If one processes this pattern by means of a multiplicative processor with the system H adjusted according to (26)

$$H = V^{-1} \tag{26}$$

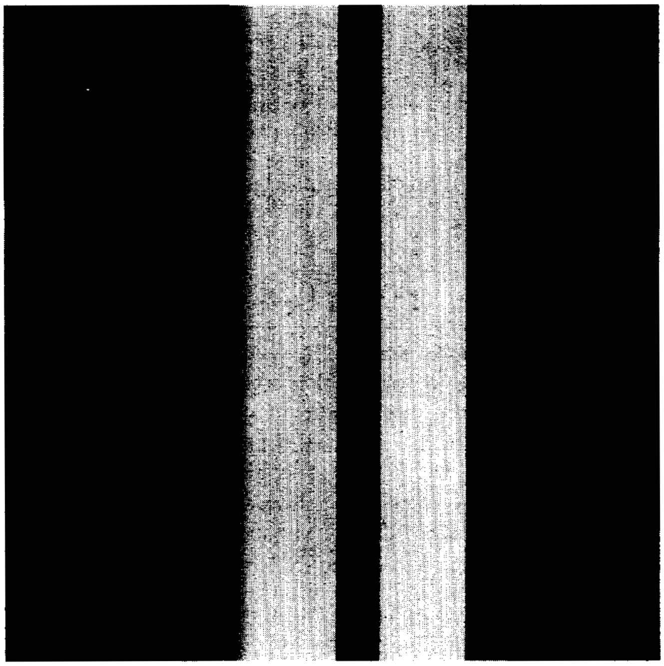
one obtains a pattern which appears to have little remaining illusion phenomena.

Such a processed pattern[14] is shown in Fig. 15. The illusions have been significantly suppressed, and the apparent brightness of Fig. 15 follows the profile of true density of Fig. 14 remarkably well. The degree to which the illusions have been suppressed provides additional support for the model of Fig. 10. In addition an estimate of the system V results as a byproduct since (26) can be solved for V in terms of the actual H used in the experiment.

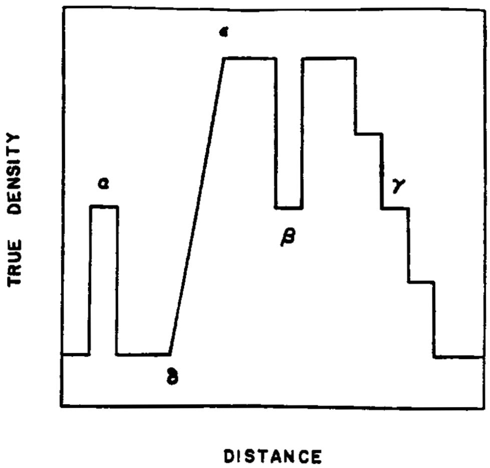
It should be noted that the above results support the logarithmic component of the model and its position in the system because the cancellation of the illusions depends upon the neutralization of the exponential component of the multiplicative processor. Without this neutralization Fig. 12(a) could not be reduced to Fig. 12(b).

Although one might find a system H that would cancel the illusions for a single fixed pattern, it has been shown that the experiment succeeds about equally well for all patterns such

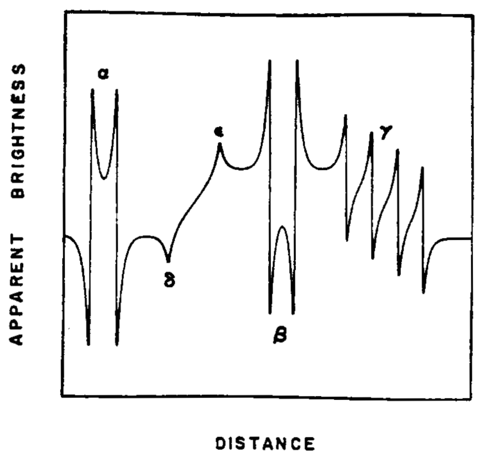
[14] Here the comments of footnote 8 must be considered most seriously since the illusion cancelling experiment is a sensitive one and gray-scale distortions can upset it easily. The calibrated print sent to the publisher appears as described in the text. A limited number of such calibrated prints are available to readers with sufficient interest and requirements. As published here the pattern should be viewed approximately at arms length.



(a)

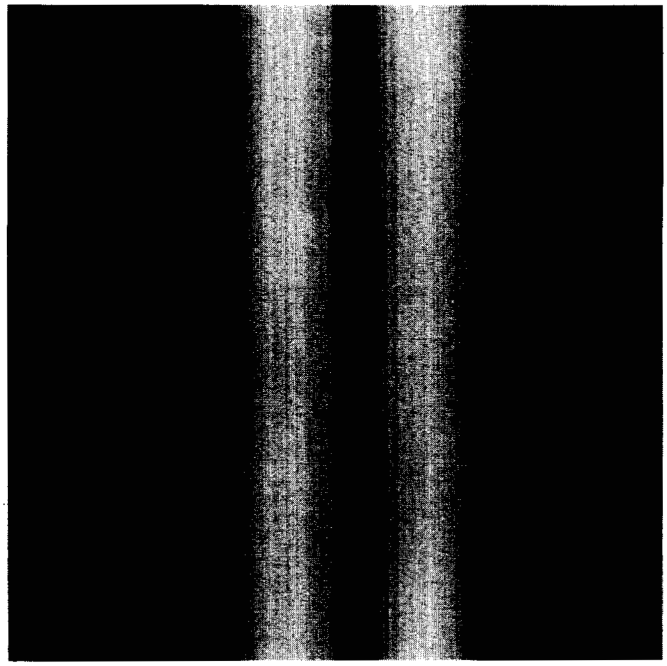


(b)

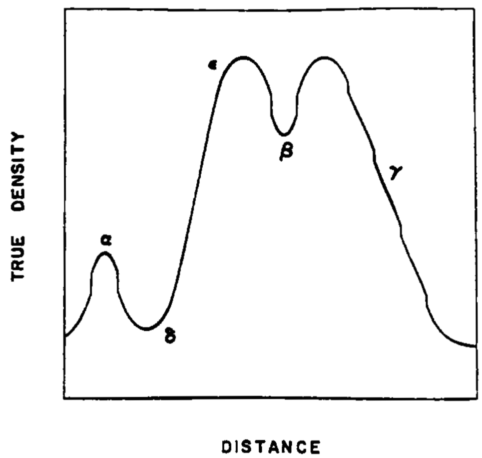


(c)

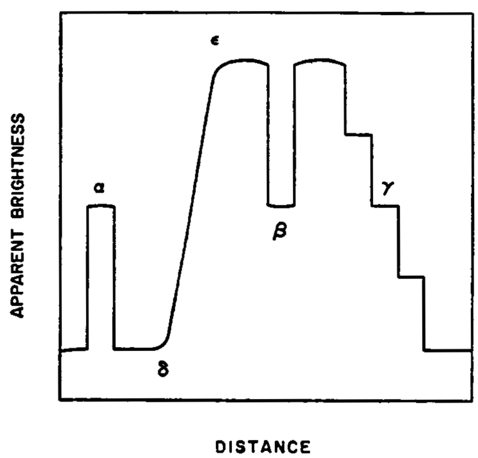
Fig. 14. Pattern for use in testing and calibrating the visual model. (a) Observe the illusions of simultaneous contrast α, β, γ, and Mach bands δ, ϵ. (b) The true density representation of the image. (c) The approximate apparent brightness of the image.



(a)

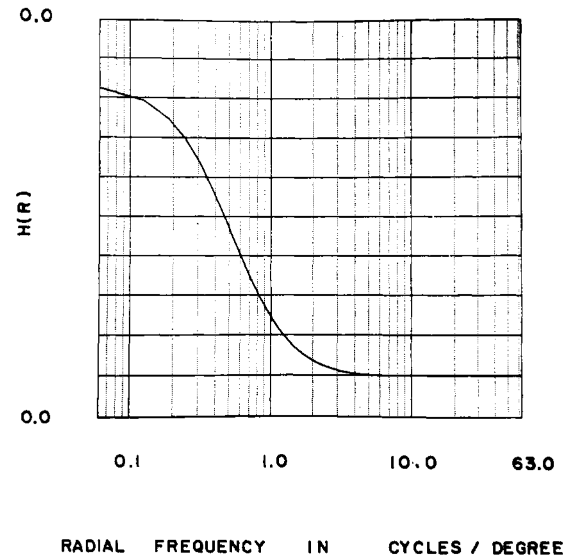


(b)

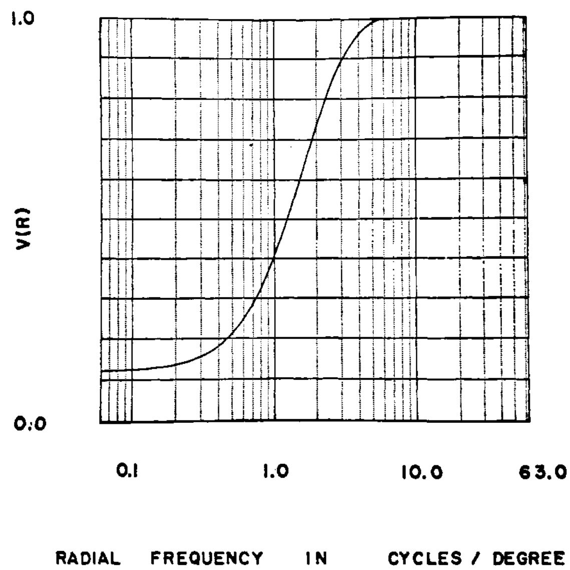


(c)

Fig. 15. The pattern of Fig. 14 processed for the suppression of optical illusions. Compare with Fig. 14. (a) Appraise the amounts of remaining simultaneous contrast α, β, γ, and Mach bands δ, ϵ. (b) The true density representation of the processed image. (c) The approximate apparent brightness of the processed image as observed from a calibrated print. Curve taken as a subjective consensus from five knowledgeable observers.



(a)



(b)

Fig. 16. Frequency response of one-dimensional systems used in test of eye model. (a) Response of system H for cancelling illusions. (b) Relative response of system V as estimated from H.

as Fig. 14 not just the one shown here. Alternately, it has been shown that the cancellation of Fig. 15 holds across a wide range of the constant of proportionality g in which the processed patterns have enough dynamic range to be clearly visible and not so much dynamic range so as to produce saturation effects.[15]

The actual linear system H used in the experiment described above was found by a cut-and-try procedure wherein an initial estimate was refined through successive rounds of processing, visual evaluation, and system redesign.

Since the test patterns varied only in one dimension, the development of a one-dimensional linear system for H was all that was required.[16] The one-dimensional frequency response of that system is shown along with its inverse in Fig. 16. It follows from two-dimensional Fourier analysis that under the assumption that the two-dimensional frequency response of the eye model has circular symmetry, the curve of Fig. 16(b) represents a radial cross section of that two-dimensional frequency response. Specifically

$$V(R) = V(X). \tag{27}$$

In addition the two-dimensional point spread function of the system V can be determined either from the Bessel transform of $V(R)$ or from the two-dimensional Fourier transform of the surface of revolution generated by $V(R)$.

It is interesting to compare the frequency response characteristics obtained here with those determined elsewhere. An excellent summary discussion and associated references are available [3, ch. 12, pp. 330–342]. In this respect there is a marked similarity between the approach taken here and the work of Davidson [3, ch. 12, pp. 330–342] in which problems with both logarithmic sensitivity and spatial interference between test patterns and matching fields are avoided.[17]

One might wonder what the world would look like if the eye did not create the illusions that we have been discussing. In this regard consider Fig. 17 which bears the same relation to Fig. 11(a) as Fig. 15(a) bears to Fig. 14(a).

VIII. Image Quality and the Visual Model

Image quality is a complicated concept and has been studied in a variety of ways and contexts. In most situations a final measure of quality can be defined only in the subjective sense. It can be measured only approximately and with difficulty by means of slow and expensive tests involving human observers. As the understanding of the human visual mechanism grows, objective measures become more feasible. So it is that with the aid of the visual model of Fig. 10 it is possible to define such a measure of image quality. By virtue of the discussions presented in Section VII one expects this measure to be related to some basic subjective considerations. An objective measure is defined by measuring the difference between a distorted image and its reference original, only after each has been transformed by the model. An example of such a definition based on a mean-square error measure is given in (28)

$$E^2 = \iint [V_{x,y} \circledast (\log I_{x,y} - \log R_{x,y})]^2 \, dxdy \tag{28}$$

[15] Since the cancellation of these illusions requires only that the apparent brightnesses of Fig. 15 take on a profile of a certain *relative* shape, the true value of g in (26) and in Fig. 13(b) cannot be determined. Thus V can only be estimated to within an unknown constant of proportionality.

[16] For the purpose of this experimental effort the linear system portion of the eye model was assumed to be position invariant. Since peripheral and central (foveal) vision possess quite different resolution properties, this assumption falls short of reality and leaves room for further refinements. For this reason and because the cancellation of illusions as shown in Fig. 15 might be improved we have not given an analytic expression for our present best estimate for $V(R)$ as part of (27). Tentatively we are using

$$V(R) = 742/(661 + R^2) - 2.463/(2.459 + R^2)$$

where R is the radial spatial frequency in cycles per degree. See Fig. 16(b). See also [7].

[17] One can still find fault with these methods, because the test patterns used do not fill the visual field and so there is still interaction between them and the surround which is uncontrolled. See also footnote 16.

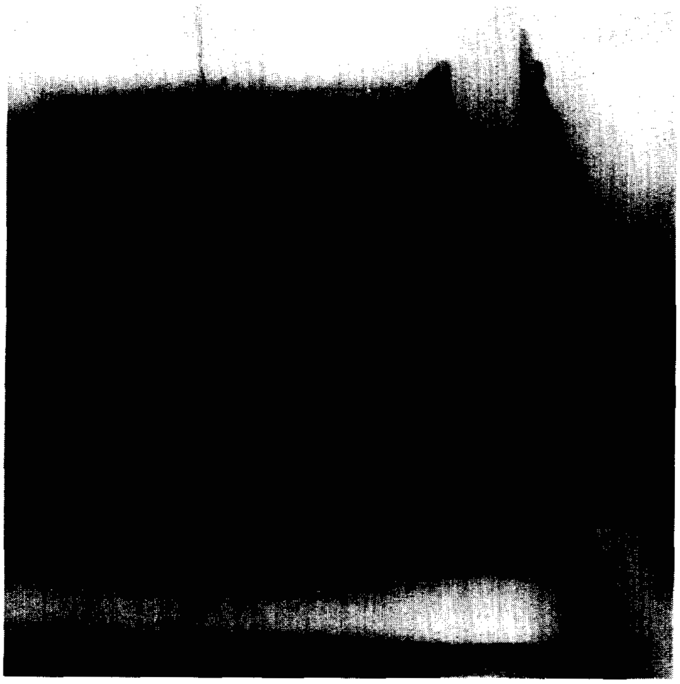


Fig. 17. The scene of Fig. 11(a) processed for the suppression of optical illusions. Compare with Fig. 11(a).

where E is the objective measure, $V_{x,y}$ is the two-dimensional point spread function of the visual model, $I_{x,y}$ is the image being measured, and $R_{x,y}$ is the reference original. For examples of the use of such an objective measure see Sakrison and Algazi [7] and Davisson [8]. Since the model emphasizes certain aspects of an image and deemphasizes certain others in a manner approximately the same as early portions of the human visual system, distortions which are important to the observer will be considered heavily while those which are not will be treated with far less weight. This will be so even though the important distortions may be physically small and the unimportant ones physically large, which is frequently the case.

With the above ideas in mind it becomes clear that when an image is to be distorted as a result of the practical limitations which characterize all transmission, storage, and processing mechanisms it makes sense to allow such distortions to take place after the image has been transformed by the model. The image can then be transformed back again just before it is to be viewed. For example if an image bandwidth compression scheme is to be implemented it probably makes much better sense to invoke that scheme upon the model-transformed image than upon the physical intensity image. The motivations for this argument are not entirely subjective. Since the model transformation emphasizes the reflectance components and deemphasizes the illumination components of a scene, it renders that scene more resistant to disturbing influences on certain physical grounds as well, because it can be argued that the reflectance component is the more important one.

For some applications it may be inconvenient to transform an image by means of the complete visual model before exposing it to disturbing influences, because the processing power required to mechanize the linear portion of the model might be somewhat high in terms of the present technology. However, for a variety of reasons it is at least desirable to employ a density representation to provide part of the resistant effect. One reason is that no disturbance can violate the property of density processing which guarantees a realizable output. Another is that since the eye is logarithmically sensitive, it considers errors on a percentage basis. Because disturbances and distortions tend to distribute themselves uniformly throughout the range of a signal, they represent extremely large percentage distortions in the dark areas of an intensity image. To make matters worse, as can be seen from the intensity histograms of Fig. 4, dark areas are by far the most likely in intensity images.

These effects can be observed most readily when images are quantized in preparation for digital processing. The classically familiar quantization contours are most visible in the dark areas of intensity represented images but distribute nearly uniformly in density represented images. As a result, the use of a given number of bits to represent an image produces more readily observable quantization distortion in the form of contouring when an intensity rather than a density representation is employed. Indeed, for images of large dynamic range the disparity can be very great.[18]

As an illustration of the issues presented in this section consider Figs. 18 and 19. Fig. 18 shows the digital original of Fig. 11(a) in combination with white noise with a rectangular probability density function. In each of the three different combinations shown the peak signal to peak noise ratio was exactly the same namely 8:1. The noise disturbs an intensity representation in Fig. 18(a), a density representation in Fig. 18(b), and a model-processed image in Fig. 18(c). For additional discussion and examples see [6].

Fig. 19 shows another image quantized to 4 bit (i.e., 16 equally spaced levels exactly spanning the signal range). The quantization disturbs an intensity representation in Fig. 19(a), and a density representation in Fig. 19(b).

IX. Summary and Conclusions

The discussions presented in this paper concentrated upon the structure of images and the compatibility of that structure with the processes used to store, transmit, and modify them. The harmony of density representation and multiplicative processing with the physics of image formation was emphasized and special attention was drawn to the fact that early portions of the human visual system seem to enjoy that harmony. A visual model based upon these observations was introduced and a test yielding a calibration for the model was presented. Finally, an objective criterion for image quality based upon that model was offered and some examples of the use of the model for protecting images against disturbances were given.

During the past five years these concepts have been developed and employed in a continuing program of digital image processing research. Their constant use in guiding the

[18] The number of bits needed to represent an image cannot properly be determined without specifying at least the quality and character of the original, the kind of processing contemplated, the quality of the final display, the representation to be used, and the dynamic range involved. Similarly, the number of bits to be saved by using a density instead of an intensity representation given a fixed subjective distortion depends at least on the dynamic range in question. In the light of the quality obtainable with present technology the "rules of thumb" which have been popularly used in the past should be regarded with caution.

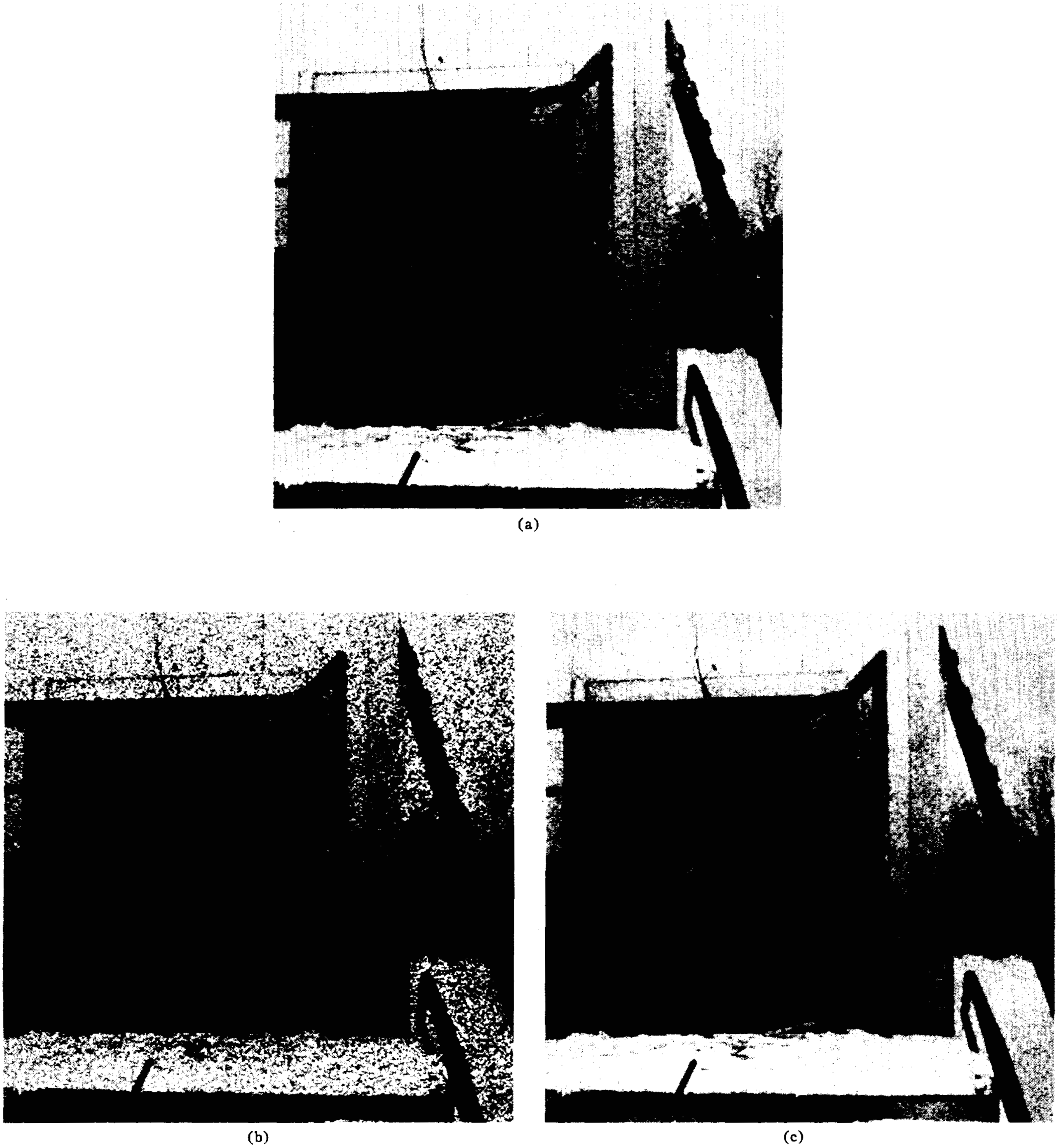
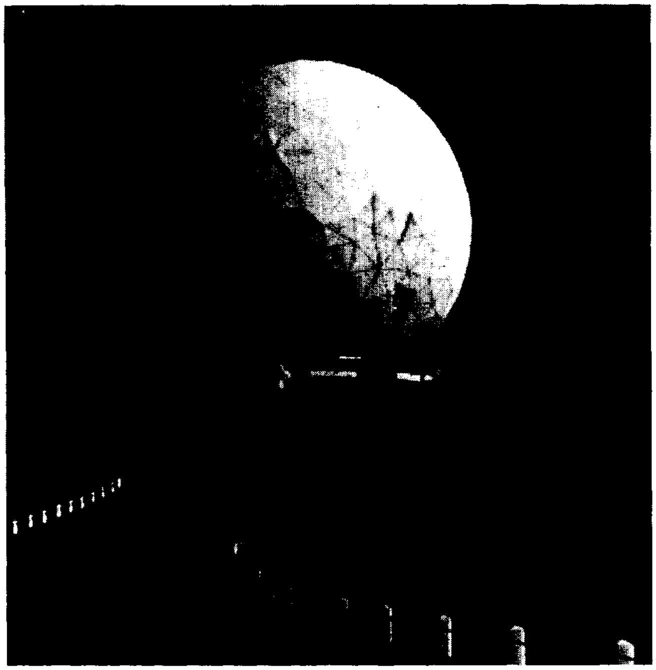


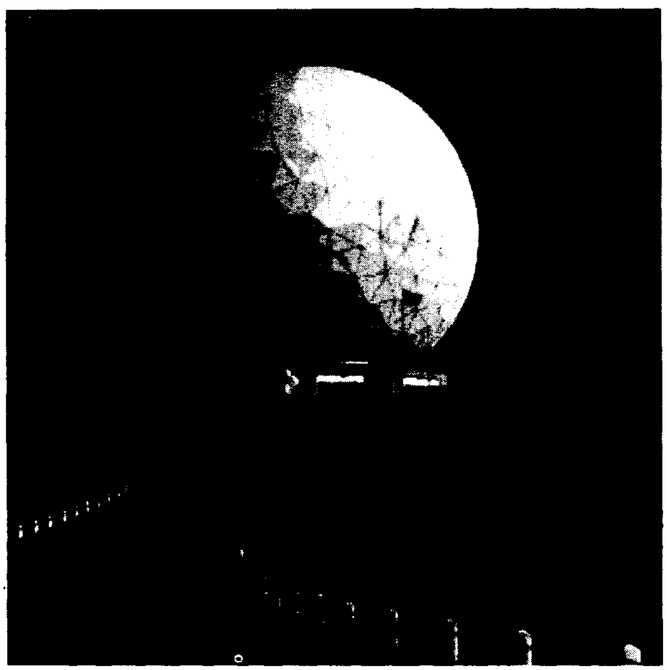
Fig. 18. Noisy disturbance in the context of three different representations. Peak signal to peak noise is 8:1 in all cases. (a) Disturbed intensities. (b) Disturbed densities. (c) Disturbed model-processed image. Compare with Fig. 11(a).

basic philosophy of the work has resulted in an ability to obtain high and consistent image quality and to enhance and simplify image processing techniques as they were proposed. Their ability to provide engineering insight and understanding complementary to existing ideas has been an invaluable aid in planning and in problem solving.

Continuing research is attempting to include within the model the aspects of color and time and to enlarge upon the model in the context of visual processes which take place at points farther along the visual pathway. It is hoped that enlargements and refinements of the model will continue to suggest useful image processing techniques and that digital



(a)



(b)

(c)

Fig. 19. Quantization distortion in the context of two different representations. In both cases 16 equally spaced levels exactly spanning the signal range were used. (a) Quantized intensities. (b) Quantized densities. (c) Original.

signal processing methods will continue to permit the investigation of those techniques which might be too complex to be explored without them.

Acknowledgment

I wish to thank the people who have helped me in the course of the image processing research which has led to the ideas presented here. I am grateful to A. V. Oppenheim for his theory of homomorphic filtering, which for me is the *sine qua non* of these views. Many thanks are also due to C. M. Ellison, D. M. Palyka, D. H. Johnson, P. Baudelaire, G. Randall, R. Cole, C. S. Lin, R. B. Warnock, R. W. Christensen, M. Milochik, Kathy Gerber, and to the many too numerous to name who have given encouragement, interest, and ideas. Special appreciation goes to my wife Martha who has given me unceasing support.

REFERENCES

[1] T. S. Huang, W. F. Schrieber, and O. J. Tretiak, "Image processing," *Proc. IEEE*, vol. 59, pp. 1586–1609, Nov. 1971.
[2] A. V. Oppenheim, R. W. Schafer, and T. G. Stockham, Jr., "Nonlinear filtering of multiplied and convolved signals," *Proc. IEEE*, vol. 56, pp. 1264–1291, Aug. 1968.
[3] T. N. Cornsweet, *Visual Perception*. New York: Academic Press, 1970.
[4] F. Ratliff, *Mach Bands: Quantitative Studies on Neural Networks in the Retina*. San Francisco, Calif.: Holden-Day, 1965.
[5] L. M. Hurvich and D. Jameson, *The Perception of Brightness and Darkness*. Boston, Mass.: Allyn and Bacon, 1966, pp. 7–9.
[6] T. G. Stockham, Jr., "Intra-frame encoding for monochrome images by means of a psychophysical model based on nonlinear filtering of multiplied signals," in *Proc. 1969 Symp. Picture Bandwidth Compression*, T. S. Huang and O. J. Tretiak, Eds. New York: Gordon and Breach, 1972.
[7] D. J. Sakrison and V. R. Algazi, "Comparison of line-by-line and two-dimensional encoding of random images," *IEEE Trans. Inform. Theory*, Vol. IT-17, pp. 386–398, July 1971.
[8] L. Davisson, "Rate-distortion theory and applications," this issue, pp. 800–808.

Image Restoration: The Removal of Spatially Invariant Degradations

MAN MOHAN SONDHI

***Abstract*—This is a review of techniques for *digital* restoration of images. Optical and other analog processors are not discussed. Restoration is considered from the point of view of space-domain as well as of spatial-frequency-domain descriptions of images. Consideration is restricted to degradations arising from noise and spatially invariant blurring. However, many of the space-domain methods apply, with minor modifications, to spatially varying blur as well. Some examples of restoration are included to illustrate the methods discussed. Included also is a section on methods whose potential has not yet been exploited for image restoration.**

I. INTRODUCTION

THE FIELD of image restoration in the modern sense of the term began in the early 1950's with the work of Maréchal and his co-workers [1]. Although the possibility of optical spatial filtering had been demonstrated by the experiments of Abbé and Porter some fifty years earlier, it was Maréchal who first recognized its potential for restoring blurred photographs. His success stimulated others to study image restoration from the point of view of optical compensation of the degradations. In the past few years the versatility of the digital computer has been brought to bear upon the problem, with promising results. With digital processing it is possible to overcome many inherent limitations of optical filtering and, indeed, to explore new approaches which have no conceivable optical counterparts.

In this paper we describe various digital techniques available for the restoration of degraded optical images. Except for references to various examples of optically restored images we exclude optical processing [2] from our discussion.

We consider imaging under incoherent illumination only and represent images by their intensity distributions. Let $p(x, y)$ represent the original undistorted picture image. We assume d to be the result of adding a noise intensity $n(x, y)$ to a blurred image $b(x, y)$ of p. We restrict our discussion to those situations where the blurring is equivalent to linear spatially invariant filtering. Thus

$$d(x, y) = b(x, y) + n(x, y) \tag{1}$$

where

$$b(x, y) = \int_{-\infty}^{\infty} dx' \int_{-\infty}^{\infty} dy' h(x - x', y - y') p(x', y'). \tag{2}$$

Here $h(x, y)$ (often called the point spread function) is the response of the blurring filter to a two-dimensional unit impulse $\delta(x)\delta(y)$.

In terms of the model of image degradation expressed by (1) and (2) we define the restoration task as follows: With d given, utilize the available *a priori* information about n, h, and p to make a good estimate $\hat{p}(x, y)$ of p. The various restoration schemes differ from each other in the assumed *a priori* information as well as in the criterion by which the goodness of the estimate is judged.

The assumption that d is available for processing is not strictly valid. Assuming instantaneous shutter action and negligible noise, the total exposure in the image plane is proportional to d. What is recorded, in general, is a nonlinear function of the exposure (e.g., the H–D curve [3] for photographic emulsions). Therefore, d may plausibly be assumed available only over a small range around the average exposure. It is possible to accurately measure the nonlinear function by using standard gray scales. Such a measurement can be used to recover d over a larger dynamic range. However, any attempt at extending this range must ultimately be frustrated by a drastic increase in the noise level.

Our assumption that noise is additive is also subject to criticism. Many of the noise sources (e.g., stray illumination, circuit noise, roundoff) may be individually modeled as additive. However, because they occur both before and after the nonlinear transduction previously mentioned their effect on d may be assumed additive only over a small dynamic range.

Manuscript received December 7, 1971; revised March 6, 1972.
The author is with Bell Telephone Laboratories, Inc., Murray Hill, N. J. 07974. Presently he is a Guest Scientist at the Department of Speech Communication, Royal Institute of Technology (KTH), Stockholm, Sweden, during the academic year 1971–1972.

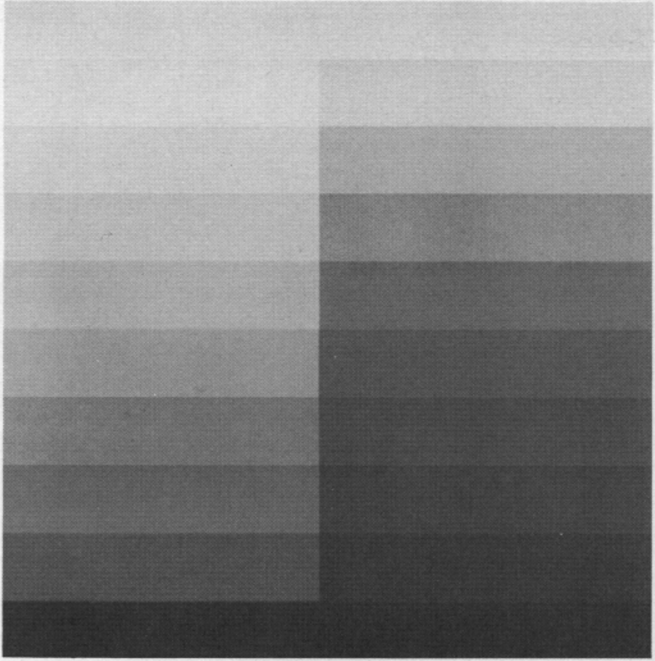


Fig. 7. Two grayscales.[8] (a) Linear intensity steps. (b) Linear density steps.

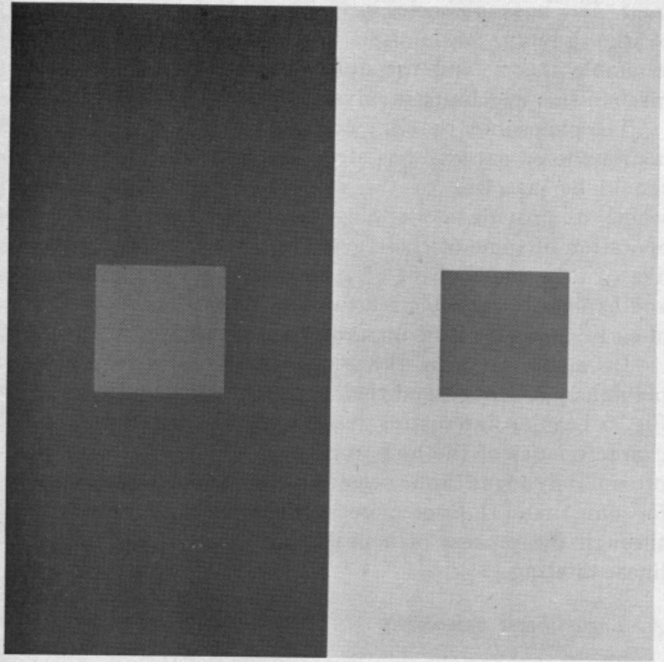
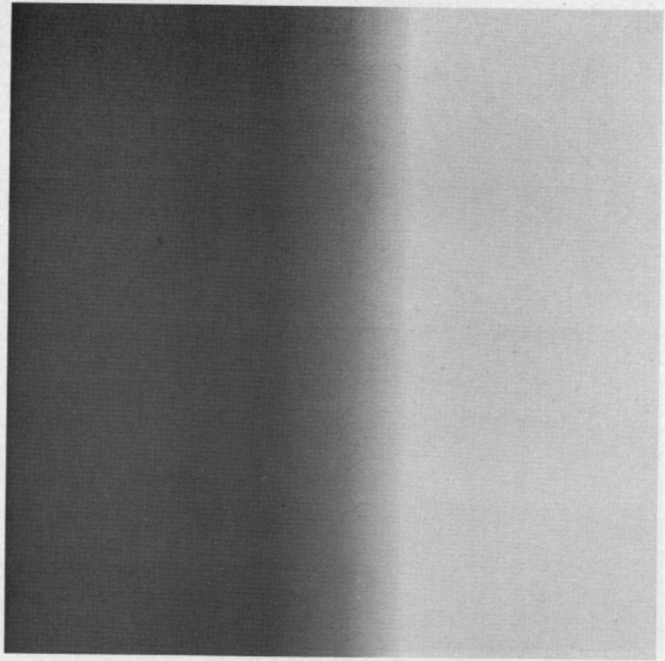
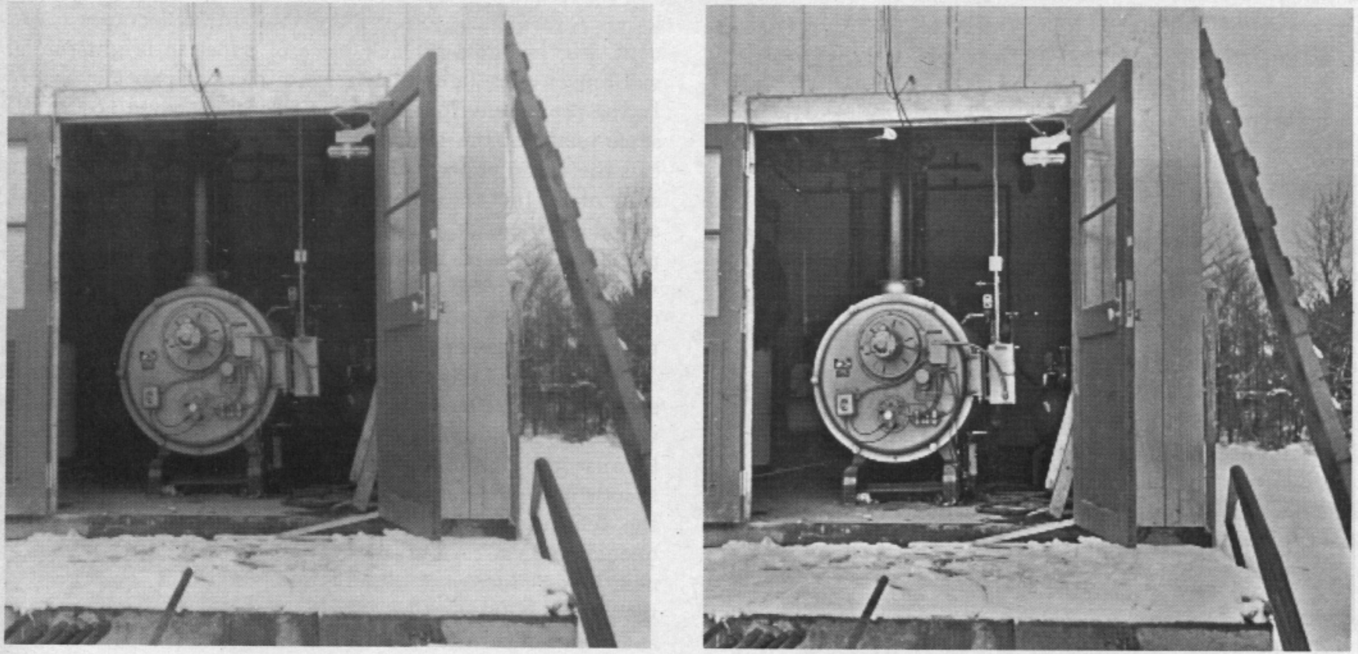


Fig. 8. The illusion of simultaneous contrast. The two small squares are of exactly the same intensity.

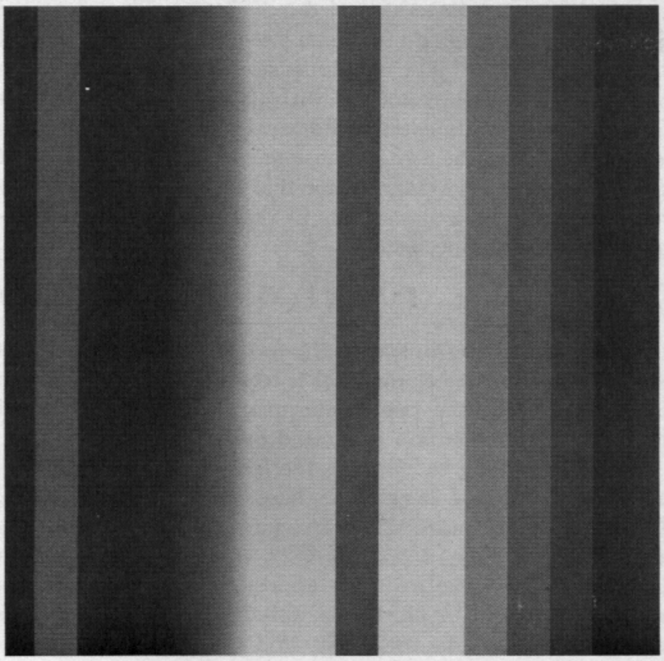


(a)

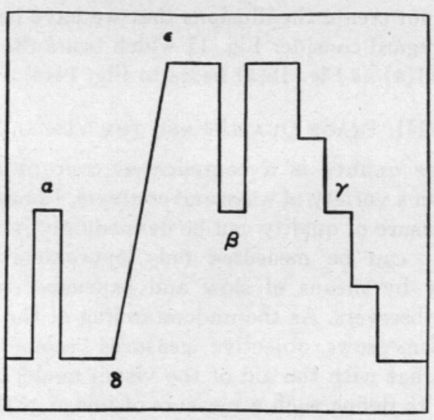


(a) (b)

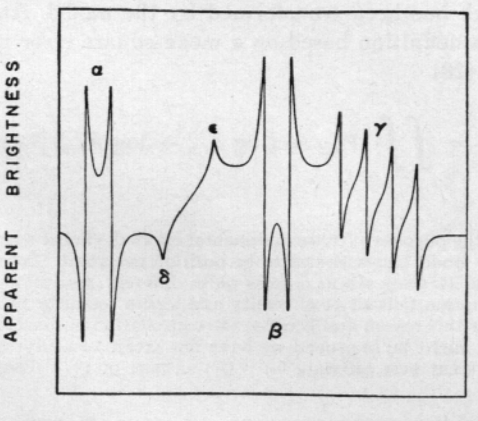
Fig. 11. A large dynamic range scene. (a) Before processing. (b) After processing with a multiplicative processor adjusted to attenuate low and to amplify high frequency components of density. (Note: These and all other images in this paper are digital.)



(a)

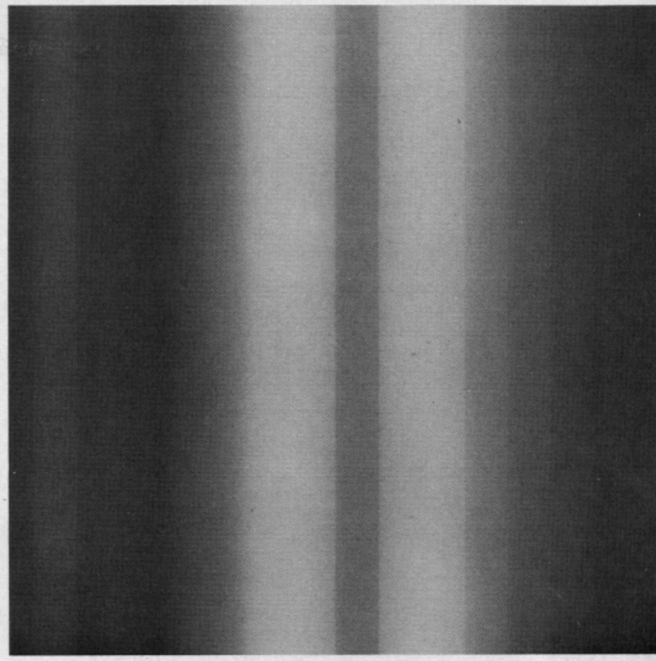


(b)

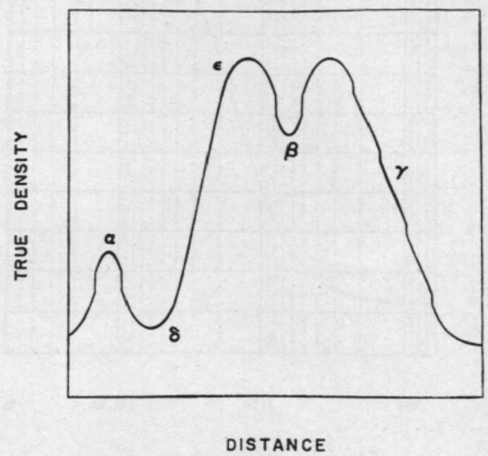


(c)

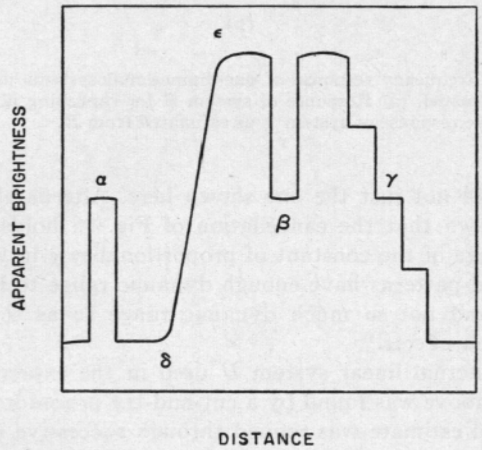
Fig. 14. Pattern for use in testing and calibrating the visual model. (a) Observe the illusions of simultaneous contrast α, β, γ, and Mach bands δ, ϵ. (b) The true density representation of the image. (c) The approximate apparent brightness of the image.



(a)



(b)



(c)

Fig. 15. The pattern of Fig. 14 processed for the suppression of optical illusions. Compare with Fig. 14. (a) Appraise the amounts of remaining simultaneous contrast α, β, γ, and Mach bands δ, ϵ. (b) The true density representation of the processed image. (c) The approximate apparent brightness of the processed image as observed from a calibrated print. Curve taken as a subjective consensus from five knowledgeable observers.

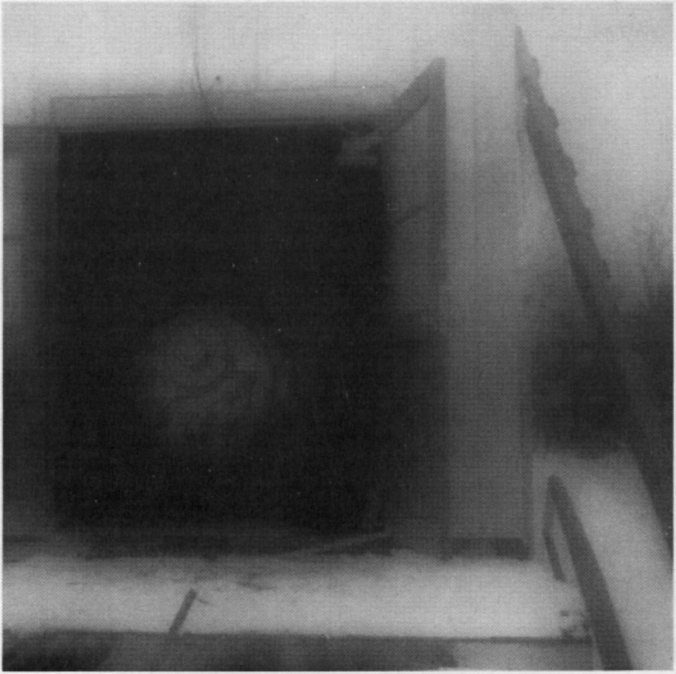
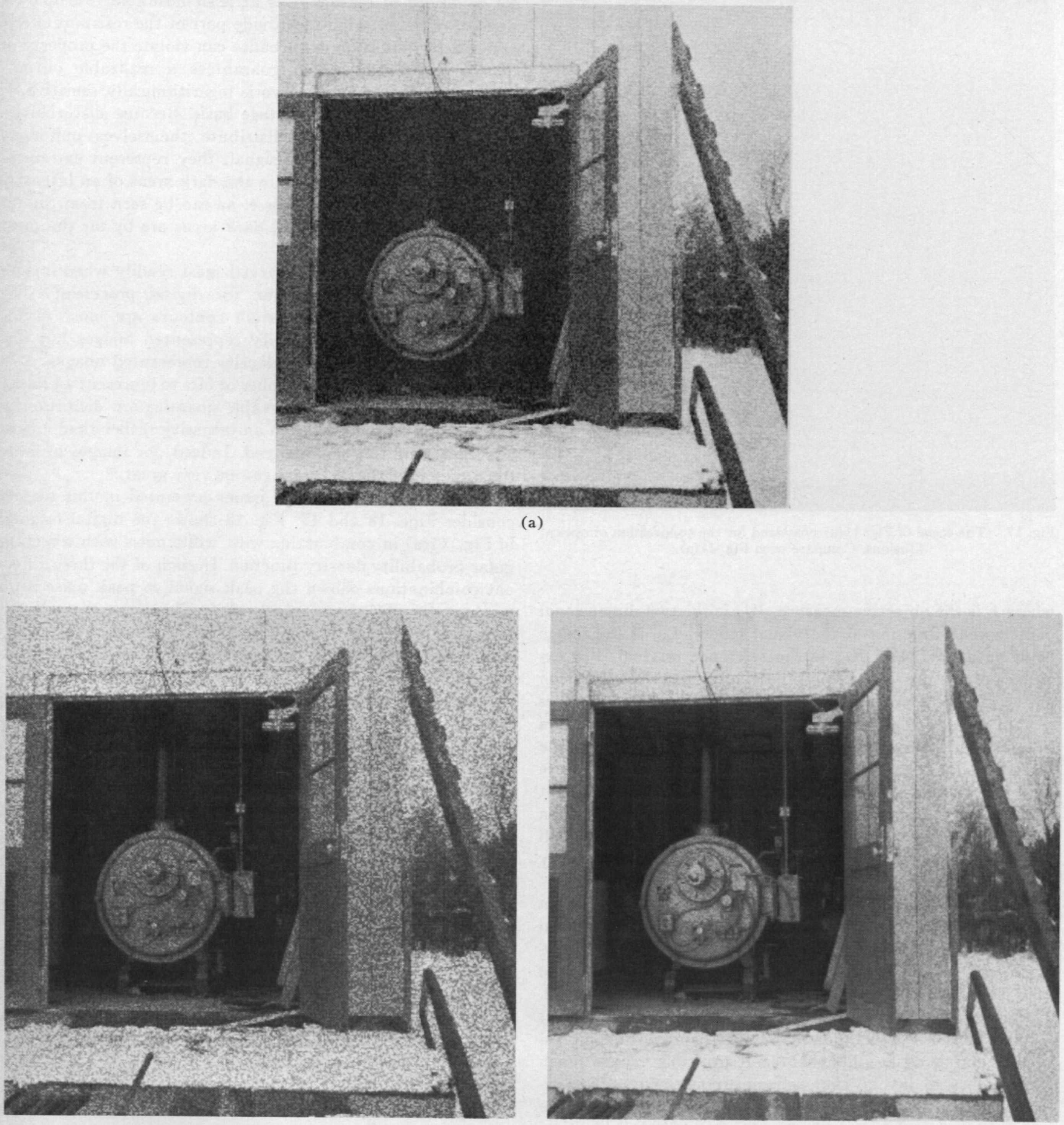


Fig. 17. The scene of Fig. 11(a) processed for the suppression of optical illusions. Compare with Fig. 11(a).



(a) (b) (c)

Fig. 18. Noisy disturbance in the context of three different representations. Peak signal to peak noise is 8:1 in all cases. (a) Disturbed intensities. (b) Disturbed densities. (c) Disturbed model-processed image. Compare with Fig. 11(a).

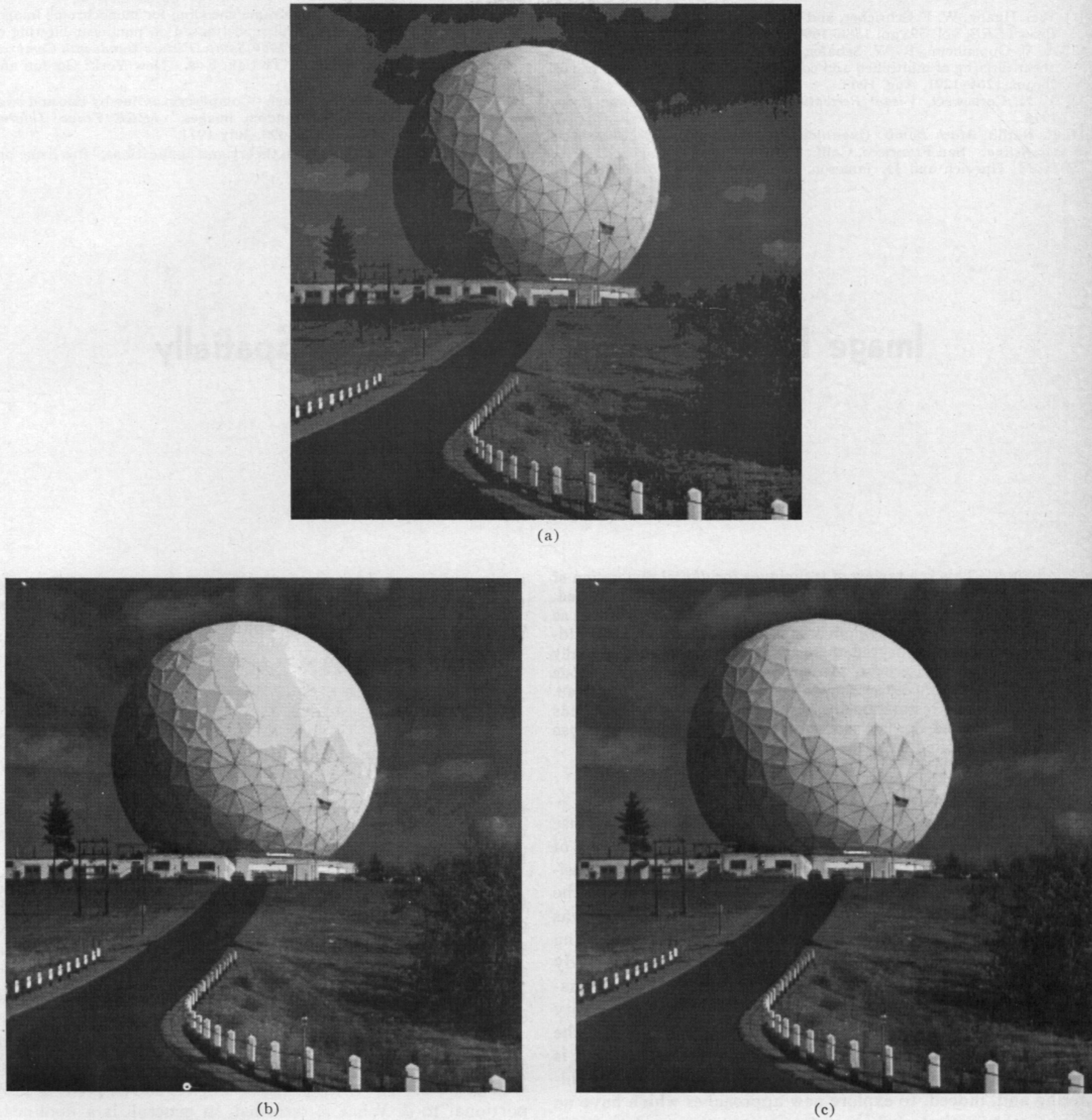


Fig. 19. Quantization distortion in the context of two different representations. In both cases 16 equally spaced levels exactly spanning the signal range were used. (a) Quantized intensities. (b) Quantized densities. (c) Original.