

Example. $f(x) = 2 * \sin(3 * \text{Pi} * x) + 10 * \sin(5 * \text{Pi} * x) + 20 * \sin(11 * \text{Pi} * x)$;
 Display the signal graph, the magnitude graph, **and** the phase graph.

Fourier transforms need the integral of $|f(x)|$ to be finite.
 Justify this detail below. Beware of trying to compute the value!

```

> u:=x->piecewise(x<0,0,1);
L:=110;
f:=unapply((5*sin(3*Pi*x)+10*sin(5*Pi*x)+14*sin(7*Pi*x)+20*sin
(11*Pi*x))*(u(x+L)-u(x-L)),x);
cv:=(1/(2*Pi)); # For Haberman Fourier transform definition
#cv:=1: # For Matlab and Maple definition (they are the same)
#cv:=1/sqrt(2*Pi): # For Asmar's definition
    u:=x->piecewise(x < 0, 0, 1)
                L:= 110
f:=x->(5 sin(3 π x) + 10 sin(5 π x) + 14 sin(7 π x)
+ 20 sin(11 π x)) (piecewise(x < -110, 0, 1) - piecewise(x < 110, 0, 1))
                cv:= 1 / (2 π)
    
```

(1)

```

> Int(abs(f(x)),x=-infinity..infinity); #evalf(%); # Finite
integral required
    
```

$$\int_{-\infty}^{\infty} \left| (5 \sin(3 \pi x) + 10 \sin(5 \pi x) + 14 \sin(7 \pi x) + 20 \sin(11 \pi x)) \left(\begin{cases} 0 & x < -110 \\ 1 & \text{otherwise} \end{cases} - \begin{cases} 0 & x < 110 \\ 1 & \text{otherwise} \end{cases} \right) \right| dx$$

(2)

```

> F:=unapply(cv*inttrans[fourier](f(x),x,-w),w); # Repaired to
match Haberman's definition
F:=w->-(I sin(110 w) (9984975 π6 - 1366877 π4 w2 + 45325 π2 w4 - 383 w6)) /
(( -w + 3 π) (w + 3 π) ( -w + 11 π) (w + 11 π) ( -w + 7 π) (w + 5 π) (w
+ 7 π) ( -w + 5 π))
    
```

(3)

Using Dirac approximation, if Dirac appears in $F(w)$
 $h := 0.8$: $\text{ApproxDirac} := x \rightarrow (1 / (2 * h)) * (\text{piecewise}(x + h < 0, 0, 1) - \text{piecewise}(x - h < 0, 0, 1))$;

```

> Mag:=unapply(abs(F(w)),w); Phase:=unapply(arctan(Im(F(w)),Re(F(w)
)),w);
Mag:=w->|(sin(110 w) (9984975 π6 - 1366877 π4 w2 + 45325 π2 w4
- 383 w6)) / (( -w + 3 π) (w + 3 π) ( -w + 11 π) (w + 11 π) ( -w + 7 π) (w
    
```

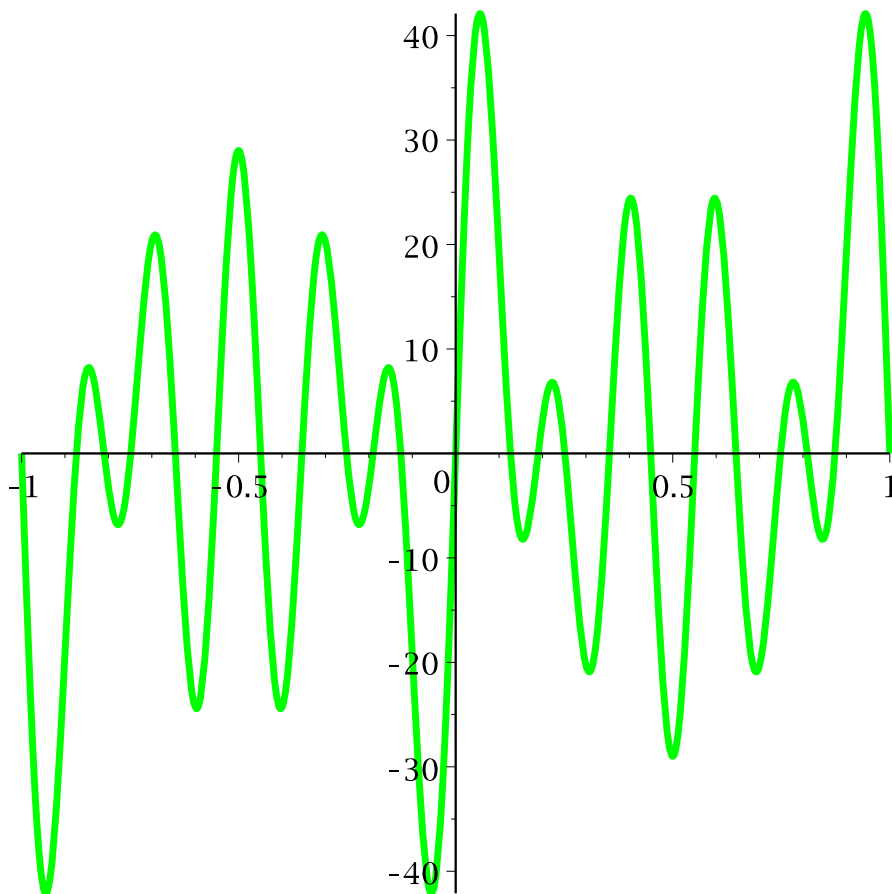
$$+ 5 \pi) (w + 7 \pi) (-w + 5 \pi)) |$$

$$\text{Phase} := w \rightarrow \arctan \left(\frac{-\Re((\sin(110 w) (9984975 \pi^6 - 1366877 \pi^4 w^2 + 45325 \pi^2 w^4 - 383 w^6)) / ((-w + 3 \pi) (w + 3 \pi) (-w + 11 \pi) (w + 11 \pi) (-w + 7 \pi) (w + 5 \pi) (w + 7 \pi) (-w + 5 \pi)))}{\Im((\sin(110 w) (9984975 \pi^6 - 1366877 \pi^4 w^2 + 45325 \pi^2 w^4 - 383 w^6)) / ((-w + 3 \pi) (w + 3 \pi) (-w + 11 \pi) (w + 11 \pi) (-w + 7 \pi) (w + 5 \pi) (w + 7 \pi) (-w + 5 \pi)))} \right) \quad (4)$$

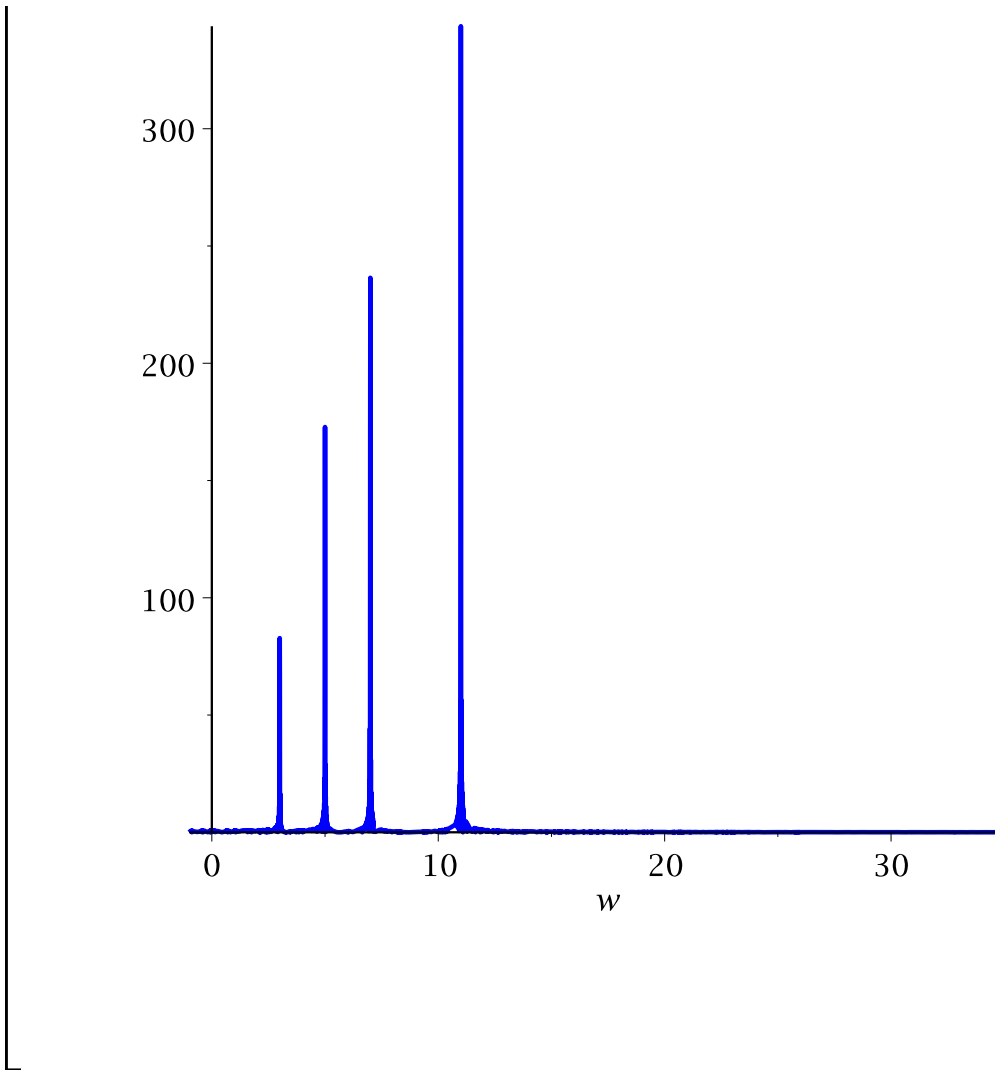
$$+ 45325 \pi^2 w^4 - 383 w^6)) / ((-w + 3 \pi) (w + 3 \pi) (-w + 11 \pi) (w + 11 \pi) (-w + 7 \pi) (w + 5 \pi) (w + 7 \pi) (-w + 5 \pi)))$$

$$\Im((\sin(110 w) (9984975 \pi^6 - 1366877 \pi^4 w^2 + 45325 \pi^2 w^4 - 383 w^6)) / ((-w + 3 \pi) (w + 3 \pi) (-w + 11 \pi) (w + 11 \pi) (-w + 7 \pi) (w + 5 \pi) (w + 7 \pi) (-w + 5 \pi)))$$

> optf:=-1..1,scaling=unconstrained,thickness=3,discont=true,color=green:plot(f,optf);



> optM:=w=-1..floor(L/Pi),scaling=unconstrained,thickness=2,color=blue:plot(Mag(w*Pi),optM);



What we see: important frequencies $3\pi, 5\pi, 7\pi, 11\pi$.
 # But the wrong magnitudes. See below.

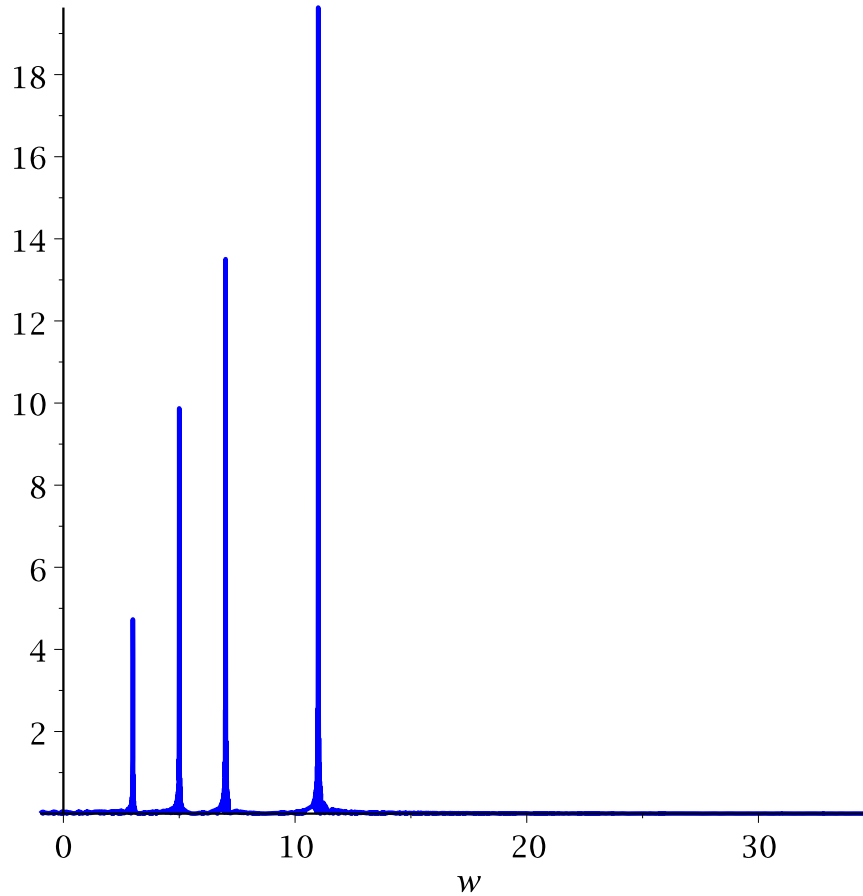
Test the equation $F(w) \cdot 2 \cdot \pi / L = a[n] + I \cdot b[n] = 2 \cdot c[n]$;

```
[ > bn:=int(f(x)*sin(11*Pi*x),x=-L..L)/L;
      bn:= 20 (5)
```

```
[ > evalf(F(11*Pi));
      evalf(F(11.000001*Pi));
      Fanswer:=limit(F(w),w=11*Pi); # Resolve bad maple answers
      Error, (in F) numeric exception: division by zero
      350.7297589 I
      Fanswer:= 1100 I (6)
                  pi
```

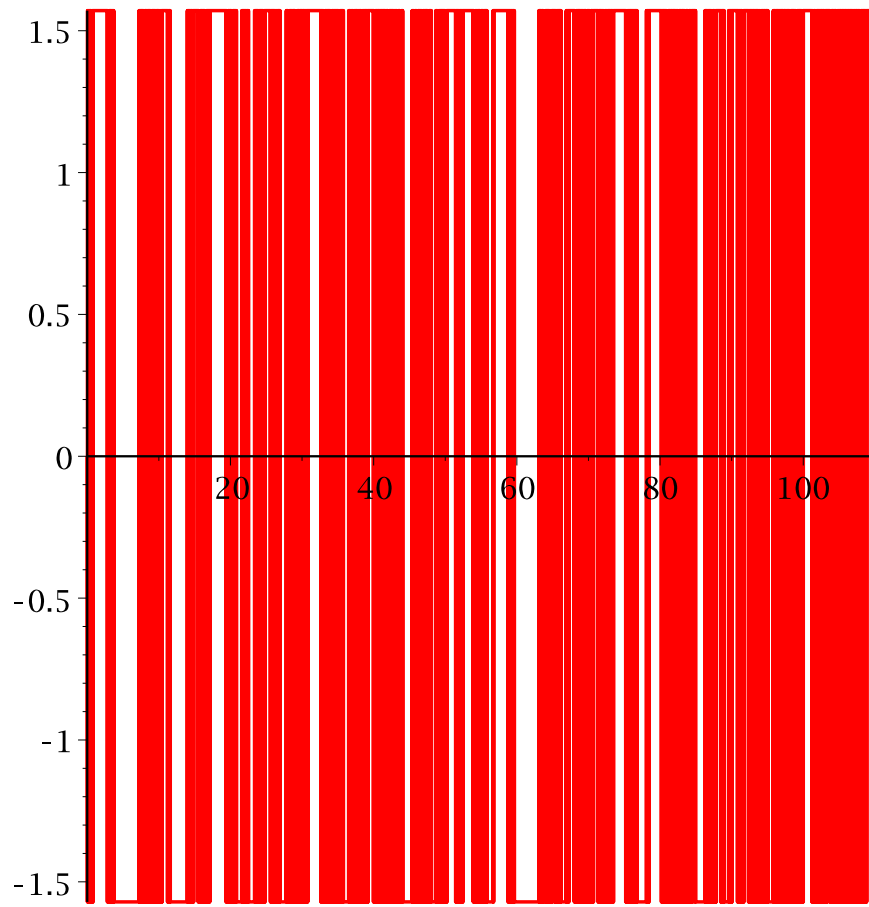
```
[ > F2:=w->abs(limit(F(x)*2*Pi/L,x=w)); # Corrected magnitude
      F2:= w -> |lim_{x->w} (2 F(x) pi / L)| (7)
```

```
> plot(F2(w*Pi),optM);
```



What we see: Correct magnitudes for the trigonometric terms, readable frequencies (no Pi factor).

```
> optP:=0..110,scaling=unconstrained,thickness=1,color=red:plot  
(Phase,optP);
```



```
> plot(Mag(w*Pi)*2*Pi/L,optM); # Works in Maple 18.
```

