```
> restart;
> #Example. f(x) = 2*sin(3*x)+10*sin(5*x)+20*sin(11*x);
>  # Display the signal graph, the magnitude graph,
>  # and the phase graph.

> # Fourier transforms need the integral of |f(x)| to be finite.
> # Justify this detail below. Beware of trying to compute the
  value!

> u:=x->piecewise(x<0,0,1);f:=unapply((2*sin(3*x)+10*sin(5*x)+20*
  sin(11*x))*(u(x)-u(x-20)),x);
  cv:=1; #cv:=1/sqrt(2*Pi):
```

$$u := x \to piecewise(x < 0, 0, 1)$$

$$f := x \to (2\sin(3x) + 10\sin(5x) + 20\sin(11x))\,(piecewise(x < 0, 0, 1) - piecewise(x < 20, 0, 1))$$

$$cv := 1 \tag{1}$$

```
> Int(abs(f(x)),x=-infinity..infinity); #evalf(%); # Need a finite
  integral.
```

$$\int_{-\infty}^{\infty} \left| (2\sin(3x) + 10\sin(5x) + 20\sin(11x)) \left(\left(\begin{cases} 0 & x < 0 \\ 1 & otherwise \end{cases} \right) - \left(\begin{cases} 0 & x < 20 \\ 1 & otherwise \end{cases} \right) \right) \right| dx \tag{2}$$

```
> F:=unapply(inttrans[fourier](cv*f(x),x,w),w);
```

$$F := w \to \frac{-276w^4 + 14856w^2 - 122100}{(w-11)(w-5)(w+11)(w+5)(w-3)(w+3)} + \frac{10\,e^{-20I(w-11)}}{w-11} + \frac{5\,e^{-20I(w-5)}}{w-5} + \frac{e^{-20I(w-3)}}{w-3} - \frac{e^{-20I(w+3)}}{w+3} - \frac{5\,e^{-20I(w+5)}}{w+5} - \frac{10\,e^{-20I(w+11)}}{w+11} \tag{3}$$

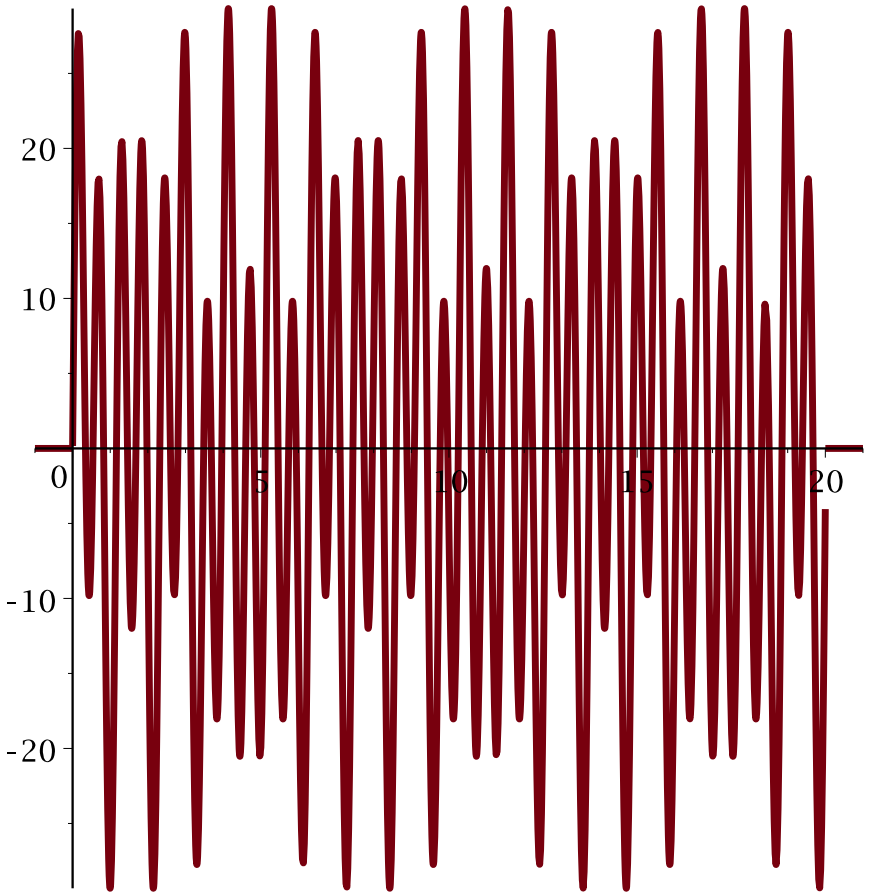
```
> # Using Dirac approximation, if Dirac appears in F(w)
  # h:=0.8:ApproxDirac:=x->(1/(2*h))*(piecewise(x+h<0,0,1)-
  piecewise(x-h<0,0,1));
> Mag:=unapply(abs(F(w)),w);
```

$$Mag := w \to \left| \frac{-276w^4 + 14856w^2 - 122100}{(w-11)(w-5)(w+11)(w+5)(w-3)(w+3)} + \frac{10\,e^{-20I(w-11)}}{w-11} + \frac{5\,e^{-20I(w-5)}}{w-5} + \frac{e^{-20I(w-3)}}{w-3} - \frac{e^{-20I(w+3)}}{w+3} - \frac{5\,e^{-20I(w+5)}}{w+5} - \frac{10\,e^{-20I(w+11)}}{w+11} \right| \tag{4}$$

```
> Phase:=unapply(arctan(Im(F(w)),Re(F(w))),w);
```

$$Phase := w \to \arctan\left(\Im\left(\frac{-276\,w^4 + 14856\,w^2 - 122100}{(w-11)\,(w-5)\,(w+11)\,(w+5)\,(w-3)\,(w+3)} + \frac{10\,e^{-20I(w-11)}}{w-11} + \frac{5\,e^{-20I(w-5)}}{w-5} + \frac{e^{-20I(w-3)}}{w-3} - \frac{e^{-20I(w+3)}}{w+3} - \frac{5\,e^{-20I(w+5)}}{w+5} - \frac{10\,e^{-20I(w+11)}}{w+11} \right), \Re\left(\frac{-276\,w^4 + 14856\,w^2 - 122100}{(w-11)\,(w-5)\,(w+11)\,(w+5)\,(w-3)\,(w+3)} + \frac{10\,e^{-20I(w-11)}}{w-11} + \frac{5\,e^{-20I(w-5)}}{w-5} + \frac{e^{-20I(w-3)}}{w-3} - \frac{e^{-20I(w+3)}}{w+3} - \frac{5\,e^{-20I(w+5)}}{w+5} - \frac{10\,e^{-20I(w+11)}}{w+11} \right) \right) \tag{5}$$

```
> optx:=-1..21,scaling=unconstrained,thickness=3,discont=true:
> optw:=-1..21,scaling=unconstrained,thickness=2,color=blue:
> plot(f,optx);plot(Mag,optw);plot(Phase,optw);
```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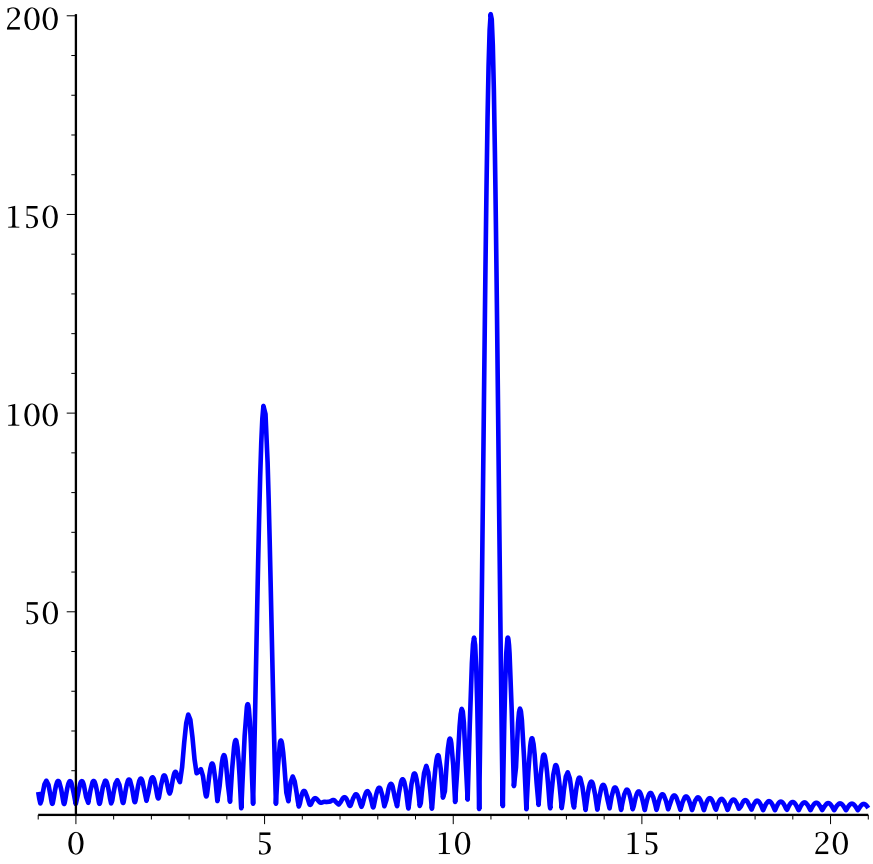
-20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100
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10
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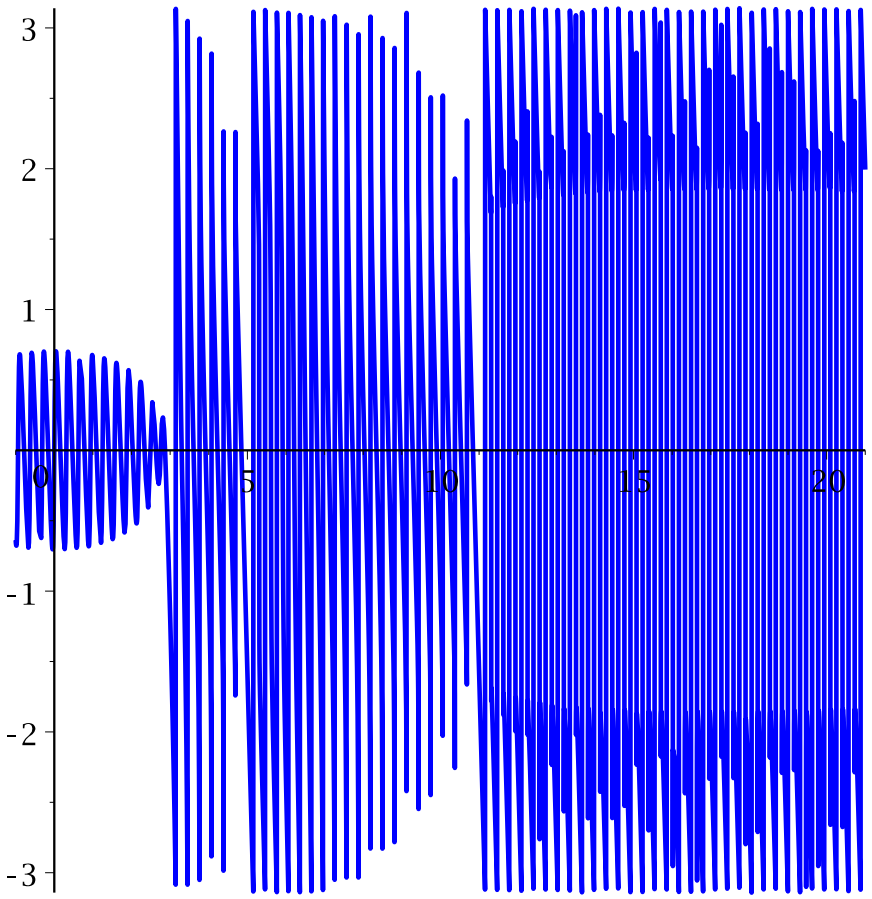


```
> 110
> 10
```