Name ___________________________

Math 3150 Problems
Haberman Chapter H3

Due Date: Problems are collected on Wednesday.

Chapter H3: 3.3 Fourier Cosine and Sine Series

Problem H3.3-1. (Sketching a Fourier Series without Computing a Series)

For the following functions, sketch $f(x)$, the Fourier series of $f(x)$, the Fourier sine series of $f(x)$, and the Fourier cosine series of $f(x)$.

(a) $f(x) = 1$

(b) $f(x) = 1 + x$

(c) $f(x) = x$ for $x < 0$ and $f(x) = 1 + x$ for $x > 0$

*(d) $f(x) = e^x$

(e) $f(x)$ =2 for $x < 0$ and $f(x) = e^{-x}$ for $x > 0$

Problem H3.3-2. (Sketch a Fourier Sine Series, Compute the Fourier Coefficients)

For the following functions, sketch the Fourier sine series of $f(x)$ and determine its Fourier coefficients.

(a) $f(x) = \cos(\pi x/L)$ Ref: Equation (13) in section H3.3

(b) $f(x) = 1$ for $x < L/6$, $f(x) = 3$ for $L/6 < x < L/2$,$f(x)$ =0 for $x > L/2$

(c) $f(x)$ =x for $x > L/2$ and zero otherwise

*(d) $f(x)$ =1 for $x < L/2$ and zero otherwise

Problem H3.3-3. (Sketch a Fourier Sine Series, Sketch a Truncated Series)

For the following functions, sketch the Fourier sine series of $f(x)$. Also, roughly sketch the sum of a finite number of nonzero terms (at least the first two) of the Fourier sine series:

(a) $f(x) = \cos(\pi x/L)$ [Use formula (13) in section H3.3]

(b) $f(x) = 1$ for $x < L/2$ and zero otherwise

(c) $f(x) = x$ [Use formula (12) in section H3.3]

Problem H3.3-4. (Sketch a Fourier Cosine Series)

Sketch the Fourier cosine series of $f(x) = \sin(\pi x/L)$. Briefly discuss.

Problem XC-H3.3-5. (Sketch a Fourier Cosine Series, Compute the Fourier Coefficients)

For the following functions, sketch the Fourier cosine series of $f(x)$ and determine its Fourier coefficients:

(a) $f(x) = x^2$

(b) $f(x) = 1$ for $x < L/6$, $f(x) = 3$ for $L/6 < x < L/2$, $f(x) = 0$ for $x > L/2$

(c) $f(x) = x$ for $x > L/2$ and zero otherwise

Problem H3.3-6. (Sketch a Fourier Cosine Series, Sketch Truncated Series)

For the following functions, sketch the Fourier cosine series of $f(x)$. Also, roughly sketch the sum of a finite number of nonzero terms (at least the first two) of the Fourier cosine series:

(a) $f(x) = x$ [Use formulas (22) and (23) in section H3.3]

(b) $f(x) = 1$ for $x > L/2$ and zero otherwise. [Use carefully formulas (6) and (7) in section H3.3.]

(c) $f(x) = x$ for $x < L/2$ and $f(x) = 1 + x$ for $x > L/2$. [Hint: Add the functions in parts (a) and (b).]

Problem H3.3-7. (Even and Odd Parts of a Function)

Show that e^x is the sum of an even and an odd function.

Problem XC-H3.3-8. (Even Extension, Odd Extension, Even Part, Odd Part)

(a) Determine formulas for the even extension of any $f(x)$. Compare to the formula for the even part of $f(x)$.

(b) Do the same for the odd extension of $f(x)$ and the odd part of $f(x)$.

(c) Calculate and sketch the four functions of parts (a) and (b) if $f(x)$ =x for $x > 0$ and $f(x) = x^2$ for $x < 0$. Graphically add the even and odd parts of $f(x)$. What occurs? Similarly, add the even and odd extensions. What occurs then?

Problem XC-H3.3-9. (Even and Odd Extensions of a Function)

What is the sum of the Fourier sine series of f(x) and the Fourier cosine series of $f(x)$? [What is the sum of the even and odd extensions of $f(x)$?]

Problem H3.3-10. (Even and Odd Parts of a Function)

If $f(x) = x^2$ for $x < 0$ and $f(x) = e^{-x}$ for $x > 0$, then what are the even and odd parts of $f(x)$?

Problem XC-H3.3-11. (Sketch Even and Odd Parts of a Function)

Given a sketch of $f(x)$, describe a procedure to sketch the even and odd parts of $f(x)$.

Problem XC-H3.3-12. (Even and Odd Terms in a Fourier Series)

(a) Graphically show that the even terms (n even) of the Fourier sine series of any function on $0 < x < L$ are odd (antisymmetric) around $x = L/2$.

(b) Consider a function $f(x)$ that is odd around $x = L/2$. Show that the odd coefficients (n odd) of the Fourier sine series of $f(x)$ on $0 < x < L$ are zero.

Problem H3.3-13. (Even Functions and Zero Terms in a Fourier Sine Series)

Consider a function $f(x)$ that is even around $x = L/2$. Show that the even coefficients (n even) of the Fourier sine series of $f(x)$ on $0 < x < L$ are zero.

Problem XC-H3.3-14. (Even Functions and Zero Terms in a Fourier Cosine Series)

(a) Consider a function $f(x)$ that is even around $x = L/2$. Show that the odd coefficients (n odd) of the Fourier cosine series of $f(x)$ on $0 < x < L$ are zero.

(b) Explain the result of part (a) by considering a Fourier cosine series of $f(x)$ on the interval $0 < x < L/2$.

Problem XC-H3.3-15. (Odd Functions and Zero Terms in a Fourier Cosine Series)

Consider a function $f(x)$ that is odd around $x = L/2$. Show that the even coefficients (n even) of the Fourier cosine series of $f(x)$ on $0 < x < L$ are zero.

Problem H3.3-16. (Fourier Series on $a < x < b$)

Fourier series can be defined on other intervals besides $-L < x < L$. Suppose that $g(y)$ is defined for $a < y < b$. Represent $g(y)$ using periodic trigonometric functions with period $b - a$. Determine formulas for the coefficients. [Hint: Use the linear transformation $y = (1/2)((a + b) + (b - a)x/L)$]

Problem XC-H3.3-17. (Arctan Function as an Integral)

Consider the integral over $x = 0$ to $x = 1$ of $1/(1 + x^2)$.

(a) Evaluate the integral explicitly.

(b) Use the Taylor series of $1/(1 + x^2)$ (itself a geometric series) to obtain an infinite series for the integral.

(c) Equate part (a) to part (b) in order to derive a formula for π.

Problem XC-H3.3-18. (Fourier Convergence for Sine and Cosine Series)

For continuous functions,

(a) Under what conditions does $f(x)$ equal its Fourier series for all x, $-L < x < L$?

(b) Under what conditions does $f(x)$ equal its Fourier sine series for all x, $0 < x < L$?

(c) Under what conditions does $f(x)$ equal its Fourier cosine series for all x, $0 < x < L$?