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Partial Differential Equations 3150

Midterm Exam 1

Exam Date: Tuesday, 27 October 2009

Instructions: This exam is timed for 50 minutes. You will be given double time to complete the exam. No calculators, notes, tables or books. Problems use only chapters 1 and 2 of the textbook. No answer check is expected. Details count 3/4, answers count 1/4.

1. (Vibration of a Finite String)
Some **normal modes** for the string equation $u_{tt} = c^2 u_{xx}$ are given by the equation

$$u(x,t) = \sin\left(\frac{n\pi x}{L}\right)\sin\left(\frac{n\pi ct}{L}\right).$$

(a) [15%] Let $c > 0$ be given and $L = 1$. Give an example of a finite linear combination of two normal modes.

(b) [25%] Explain why the example given in (a) is a solution of $u_{tt} = c^2 u_{xx}$. You may cite textbook results to simplify the explanation.

(c) [60%] Solve the finite string vibration problem on $0 \le x \le 1$, $t > 0$,

$$\begin{aligned} u_{tt} &= c^2 u_{xx}, \\ u(0,t) &= 0, \\ u(1,t) &= 0, \\ u(x,0) &= 0, \\ u_t(x,0) &= 7\sin(\pi x) + 4\sin(5\pi x) + 3\sin(11\pi x). \end{aligned}$$

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2. (Periodic Functions)

(a) [25%] Find the fundamental period of $f(x) = \sin 2x + \cos 3x$.

(b) [25%] Give an example of a piecewise smooth function on $-1 \le x \le 2$ that has discontinuities at $x = 0$ and $x = 1$.

(c) [25%] Find all values of constant k such that $f(x) = \cos(2x + k)$ is an even periodic function.

(d) [25%] The odd function $f(x) = \sin \pi x$ is positive on $0 < x < 1$. It can be rectified to an even periodic function $g(x)$ on $-1 < x < 1$. Write a formula for $g(x)$.

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3. (Fourier Series)
Let $f(x) = -2$ on the interval $0 < x < 2\pi$, $f(x) = 2$ on $-2\pi < x < 0$, $f(x$ left undefined for the moment at $x = 0, 2\pi, -2\pi$. There are many 4π-periodic extensions of f to the whole real line. Let $g(x)$ denote any one such extension, obtained by some appropriate definition of $f(x)$ at $x = 0, 2\pi, -2\pi$.

(a) [25%] Does $g(x)$ have to be even or odd? Explain your answer.

(b) [25%] Display the formulas for the Fourier coefficients of f.

(c) [25%] Compute the Fourier coefficient for the term $\sin(10x)$.

(d) [25%] Assume $g(x)$ equals the Fourier series of f for every x in $-\infty < x < \infty$. Find the values of $g(x)$ at $x = 0, 2\pi, -2\pi$.

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4. (Cosine and Sine Series)
Find the first eight terms in the sine series expansion of the sine wave $g(x)$, formed as the odd periodic extension of the base function $\sin 2x + 2\sin 6x$ on $0 < x < \pi$.

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5. (Convergence of Fourier Series)

(a) [25%] State the Fourier Convergence Theorem for piecewise smooth functions.

(b) [75%] Fourier convergence may not be uniform, and the commonly referenced term to describe this problem is Gibb's phenomenon. Explain what it is, via the example $f(x) = -1$ on $-1 < x < 0$, $f(x) = 2$ on $0 < x < 2$, and its 3-periodic extension $g(x)$ to $-\infty < x < \infty$.

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